

On the distributional effects of commodity taxation

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Abstract

A commodity tax system is inequality reducing if the after-tax distribution of income Lorenz dominates the before-tax distribution of income, regardless of initial conditions. This paper identifies necessary and sufficient conditions under which an ad valorem commodity tax system is inequality reducing, shedding light on the role of taxing luxury—as opposed to necessary—commodities in the equalization of after-tax incomes.

KEYWORDS

D63, D71, commodity taxation, income inequality, Lorenz domination, progressive taxation

1 | INTRODUCTION

The study of indirect taxation has traditionally revolved around the efficiency properties of commodity tax systems. There is an extensive literature on optimal indirect taxation, which dates back to the seminal work of Ramsey (1927).¹ The conditions under which a commodity tax system minimizes its associated deadweight loss are well understood, but there is very little work dealing with the distributional effects of commodity taxation as they pertain to income inequality. Some authors have advocated consumption taxes as a means to encourage saving, arguing that a progressive consumption tax—whereby each household is taxed on its consumption, not income, at graduated rates—can mimic the distribution of the tax burden in the current income tax (Seidman, 2003). Other authors have justified differential tax rates across goods on grounds of equity, making a case for lower tax rates on goods consumed disproportionately by low-income households (Mirrlees et al., 2011, Chapter 6).² The

¹See, for example, Salanié (2003) and references therein.

²The discussion in Mirrlees et al. (2011, Chapter 6) (see, in particular, their Section 6.2) propounds equity and efficiency arguments both for and against tax uniformity. While equity principles favoring differentiated taxation—as a means of redistribution—are emphasized when indirect taxation is considered in isolation, the takeaway from the general discussion in Mirrlees et al. (2011, Chapter 6) is that indirect taxation should be uniform and that redistributive goals should be left for direct tax and transfer policies.

discussion, however, is kept at an informal level, and can hardly be seen as a rigorous foundation for the income inequality effects of commodity taxation.

This paper is an attempt to understand the conditions under which commodity taxation reduces the inequality of the after-tax income distribution. The analysis conducted here is akin to that underlying the normative foundation for progressive income tax schedules based on the relative Lorenz ordering, initiated by Jakobsson (1976) and Fellman (1976), and extended by other authors in several directions (see, inter alia, Carbonell-Nicolau & Llavador, 2018, 2019a, 2019b; Ebert & Moyes, 2000, 2007; Eichhorn, Funke, & Richter, 1984; Formby, James Smith, & Sykes, 1986; Hemming & Keen, 1983; Ju & Moreno-Tertero, 2008; Kakwani, 1977; Latham, 1988; LeBreton, Moyes, & Trannoy, 1996; Liu, 1985; Moyes, 1988, 1994; Preston, 2007; Thistle, 1988; Thon, 1987)

The paper confines attention to the linear case—the Ramsey setting—and leaves the case of “mixed taxation,” covering the combined effects of commodity and (nonlinear) income taxation, for future research. The received wisdom from the literature on optimal taxation is the Atkinson–Stiglitz principle (Atkinson & Stiglitz, 1976), which asserts that, under separability of utility between labor and consumption, indirect taxation is superfluous. Some remarks as to why the Atkinson–Stiglitz principle is likely to fail in the present framework are furnished in Section 4.

Section 2 presents a preliminary analysis of the case when pre-tax incomes are unresponsive to taxation. Given an initial distribution of incomes, a commodity tax system gives rise to an after-tax distribution of disposable incomes. Theorem 1 identifies necessary and sufficient conditions on primitives under which a commodity tax system reduces income inequality—in the sense of the relative Lorenz ordering—regardless of the initial income distribution it is applied to. These conditions shed light on the distributional effects of taxing *luxury goods*—as opposed to *necessary goods*. In particular, any *progressive* commodity tax system that taxes luxuries and (weakly) subsidizes necessities leads to a decline in inequality (Corollary 1).

Section 3 extends the analysis to the case of endogenous income. Theorem 2 characterizes inequality reducing commodity tax systems in the presence of potential effects of taxes on labor supply. This result reveals that the condition defining luxuries (resp., necessities) is no longer the sole determinant of the income inequality effects of commodity taxation: the wage elasticity of (before-tax) income also plays an important role. In particular, the effect of a luxury tax on this elasticity might counter the induced bias of the tax burden towards the rich. Section 3 develops intuition for the main characterization in Theorem 2, while the formal proof is relegated to the appendix. A number of applications illustrate Theorem 2.

2 | EXOGENOUS INCOME

Even though the full-fledged model (developed in Section 3) encompasses the case of exogenous income, it is useful to begin with the analysis of this case.

Consumer behavior is described by means of a social utility function $u: \mathbb{R}_+^K \rightarrow \mathbb{R}$, defined over consumption bundles \mathbf{x} , where $\mathbf{x} = (x_1, \dots, x_K)$ is a bundle of $K \geq 1$ traded goods. It is assumed that $u(\mathbf{x})$ is strictly increasing, continuous, and strictly quasi-concave.³

Let \mathcal{U} denote the collection of all utility functions satisfying the above conditions.

³The results of the paper hold intact if one considers instead utility functions of the Cobb–Douglas type, or quasi-linear utility functions, which fail strong monotonicity and strict quasi-concavity on (some of) the axes.

Definition 1. A commodity tax system is a vector $\tau = (\tau_1, \dots, \tau_K) \in \mathbb{R}^K$ of tax rates, one for each traded good. For each good k , the system levies the amount τ_k per unit of good k consumed, if $\tau_k > 0$, while $\tau_k < 0$ is interpreted as a subsidy.

Given a commodity tax system τ and a commodity price vector $\mathbf{p} = (p_1, \dots, p_K) \gg 0$ such that $p_k + \tau_k > 0$ for each k , an agent with income y solves the following problem:

$$\begin{aligned} & \max_{\mathbf{x} \in \mathbb{R}_+^K} u(\mathbf{x}), \\ & \text{s.t.}, \\ & (\mathbf{p} + \boldsymbol{\tau})\mathbf{x} := (p_1 + \tau_1)x_1 + \dots + (p_K + \tau_K)x_K \leq y. \end{aligned} \tag{1}$$

Because u is continuous and strictly quasi-concave, (1) has a unique solution. Given $u \in \mathcal{U}'$, the associated Marshallian demand functions for the K commodities are denoted by

$$X_1^u(\mathbf{p}, y), \dots, X_K^u(\mathbf{p}, y),$$

where $\mathbf{p} \gg 0$ is the price vector and y represents income. The solution to (1) is given by

$$(X_1^u(\mathbf{p} + \boldsymbol{\tau}, y), \dots, X_K^u(\mathbf{p} + \boldsymbol{\tau}, y)), \tag{2}$$

where $\mathbf{p} + \boldsymbol{\tau} := (p_1 + \tau_1, \dots, p_K + \tau_K)$. The post-tax income function associated with the vector in (2) is given by

$$\begin{aligned} z^u(y, \mathbf{p}, \boldsymbol{\tau}) & := y - \tau_1 X_1^u(\mathbf{p} + \boldsymbol{\tau}, y) - \dots - \tau_K X_K^u(\mathbf{p} + \boldsymbol{\tau}, y) \\ & = p_1 X_1^u(\mathbf{p} + \boldsymbol{\tau}, y) + \dots + p_K X_K^u(\mathbf{p} + \boldsymbol{\tau}, y), \end{aligned} \tag{3}$$

that is, $z^u(y, \mathbf{p}, \boldsymbol{\tau})$ represents expenditure on goods (net of taxes). It is clear that $z^u(y, \mathbf{p}, \boldsymbol{\tau}) > 0$ whenever $y > 0$. In addition, as a consequence of Berge's Maximum Theorem, each $X_k^u(\cdot)$ is continuous, implying that $z^u(\cdot)$ is continuous.

Let \mathcal{U} denote the collection of all utility functions in \mathcal{U}' such that for any $\mathbf{p} \gg 0$, the map $y \mapsto X_k^u(\mathbf{p}, y)$, $k \in \{1, \dots, K\}$, is differentiable on \mathbb{R}_{++} .⁴ For $u \in \mathcal{U}$, $\mathbf{p}, \boldsymbol{\tau}$ (with $\mathbf{p} + \boldsymbol{\tau} \gg 0$), and $y > 0$, one has

$$\begin{aligned} \frac{\partial z^u(y, \mathbf{p}, \boldsymbol{\tau})}{\partial y} & = 1 - \tau_1 \cdot \frac{\partial X_1^u(\mathbf{p} + \boldsymbol{\tau}, y)}{\partial y} - \dots - \tau_K \cdot \frac{\partial X_K^u(\mathbf{p} + \boldsymbol{\tau}, y)}{\partial y} \\ & = p_1 \cdot \frac{\partial X_1^u(\mathbf{p} + \boldsymbol{\tau}, y)}{\partial y} + \dots + p_K \cdot \frac{\partial X_K^u(\mathbf{p} + \boldsymbol{\tau}, y)}{\partial y}. \end{aligned} \tag{4}$$

⁴The differentiability assumption is not innocuous. Katzner (1968) showed that demand functions derived from well-behaved utility functions need not be differentiable everywhere. However, Mas-Colell (1974) showed that differentiable demand functions are "generic" (see Mas-Colell, 1974 for details). In any case, here it suffices to assume that the right derivative of the map $y \mapsto X_k^u(\mathbf{p}, y)$ defined on \mathbb{R}_{++} exists at any point in \mathbb{R}_{++} . Under this weaker assumption, the statement of Theorem 1 should be modified as follows: " $\partial z^u(y, \mathbf{p}, \boldsymbol{\tau})/\partial y$ " and " $\partial X_k^u(\mathbf{p} + \boldsymbol{\tau}, y)/\partial y$ " should be replaced by their corresponding right partial derivatives.

Note that, if at least one good is inferior, there is no guarantee that disposable (after-tax) incomes are nondecreasing in income, that is, that $(\partial z^u(y, \mathbf{p}, \tau)/\partial y) \geq 0$ holds for all (y, \mathbf{p}, τ) .

Given a fixed (but otherwise arbitrary) population size $n \in \mathbb{N}$, an *income distribution* is a vector $\mathbf{z} = (z_1, \dots, z_n) \in \mathbb{R}_+^n$ of incomes with $\max_i z_i > 0$. Here, z_i denotes the income of agent i . Given an income distribution $\mathbf{z} = (z_1, \dots, z_n)$, let $(z_{[1]}, \dots, z_{[n]})$ be a rearrangement of the coordinates in \mathbf{z} with the property that $z_{[1]} \leq \dots \leq z_{[n]}$.

In this paper, the standard relative Lorenz ordering is used to make inequality comparisons between income distributions. The focus here is on *income* (as opposed to *welfare*) inequality comparisons, as in virtually all the related literature. As pointed out in Carbonell-Nicolau and Llavador (2018), a characterization of the *welfare* inequality reducing properties of tax systems is likely to be problematic on account of the strong cardinal nature of the notion of welfare inequality, which is not generally invariant to order-preserving utility transformations.⁵

Given two income distributions, $\mathbf{z} = (z_1, \dots, z_n)$ and $\mathbf{z}' = (z'_1, \dots, z'_n)$, the *relative Lorenz preorder* \succeq_L is defined by

$$\mathbf{z}' \succeq_L \mathbf{z} \Leftrightarrow \left[\forall m \in \{1, \dots, n\}; \frac{\sum_{i=1}^m z'_{[i]}}{\sum_{i=1}^n z'_{[i]}} \geq \frac{\sum_{i=1}^m z_{[i]}}{\sum_{i=1}^n z_{[i]}} \right].$$

Together with a commodity tax system τ and a price vector \mathbf{p} (with $\mathbf{p} + \tau \gg 0$), a *pre-tax income distribution* $\mathbf{y} = (y_1, \dots, y_n)$ determines a *post-tax income distribution*:

$$\mathbf{z}^u(\mathbf{y}, \mathbf{p}, \tau) := (z^u(y_1, \mathbf{p}, \tau), \dots, z^u(y_n, \mathbf{p}, \tau)).$$

Definition 2. Given $u \in \mathcal{U}$ and a price vector $\mathbf{p} \gg 0$, a commodity tax system τ satisfying $\mathbf{p} + \tau \gg 0$ is *income inequality reducing with respect to* (u, \mathbf{p}) ((u, \mathbf{p}) -iir) if $\mathbf{z}^u(\mathbf{y}, \mathbf{p}, \tau) \succeq_L \mathbf{y}$ for each pre-tax income distribution \mathbf{y} .

The following result is a characterization of inequality reducing commodity tax systems. Its proof is relegated to the appendix.

Theorem 1. Given $u \in \mathcal{U}$ and a price vector $\mathbf{p} \gg 0$, a commodity tax system τ with $\mathbf{p} + \tau \gg 0$ is (u, \mathbf{p}) -iir if and only if $(\partial z^u(y, \mathbf{p}, \tau)/\partial y) \geq 0$ for all $y > 0$ and

$$\sum_{k=1}^K \tau_k \cdot \frac{X_k^u(\mathbf{p} + \tau, y)}{y} \leq \sum_{k=1}^K \tau_k \cdot \frac{\partial X_k^u(\mathbf{p} + \tau, y)}{\partial y} \text{ for all } y > 0. \quad (5)$$

Theorem 1 can be used to shed light on the role of taxing luxury—as opposed to necessary—commodities in the after-tax equalization of incomes.

Definition 3. A commodity k is called a *luxury commodity* if the proportion of total income spent on it rises with income, that is, if

⁵A different, but related, measure is the Atkinson characterization of social welfare in terms of a notion of *generalized Lorenz dominance* (see, e.g., Atkinson, 1970; Shorrocks, 1983), which is not generally equivalent to the metric used in this paper.

$$\frac{\partial(p_k X_k^u(\mathbf{p}, y)/y)}{\partial y} > 0 \quad \text{for each } (\mathbf{p}, y); \quad (6)$$

commodity k is called a *necessary commodity* if the said proportion declines with income (i.e., the inequality in (6) is reversed).

Remark 1. Note that a luxury commodity is necessarily a normal commodity, but the converse assertion does not hold in general. Also, an inferior (and, in particular, a Giffen) commodity is necessarily a necessity, but necessities may be either normal or inferior commodities.

Remark 2. Condition (6) is equivalent to

$$\frac{\partial X_k^u(\mathbf{p}, y)}{\partial y} > \frac{X_k^u(\mathbf{p}, y)}{y} \quad \text{for each } (\mathbf{p}, y), \quad (7)$$

and so a commodity is a luxury if (7) holds. Similarly, necessity is a commodity for which the same condition holds with the inequality reversed.

In light of Remark 2, Theorem 1 reveals that the condition defining luxuries (resp., necessities) has a direct bearing on the inequality reducing properties of a tax system. Specifically, taxation of luxuries (resp., necessities) leads to a lower (resp., higher) term on the left-hand side of (5), relative to the term on the right-hand side. Consequently, so-called *progressive commodity tax systems* (Definition 4) are inequality reducing.

Definition 4. A commodity tax system τ is called *progressive* if $\tau_k > 0$ whenever k is a luxury and $\tau_k \leq 0$ whenever k is a necessity.

Corollary 1 (To Theorem 1). *Given $u \in \mathcal{U}$ and a price vector $\mathbf{p} \gg 0$, any progressive commodity tax system τ with $\mathbf{p} + \tau \gg 0$ and $(\partial z^u(y, \mathbf{p}, \tau)/\partial y) \geq 0$ for all $y > 0$ is (u, \mathbf{p}) -iir.*

Remark 3. Note that, from (4), it follows that, for fixed \mathbf{p} , under uniform boundedness of the partial derivatives $\partial X_k^u(\mathbf{p}', y)/\partial y$ ($k \in \{1, \dots, K\}$) over the pairs (\mathbf{p}', y) , where \mathbf{p}' represents a slight perturbation of \mathbf{p} , and where $y > 0$, any τ sufficiently close to zero has $\partial z^u(y, \mathbf{p}, \tau)/\partial y \geq 0$ for all $y > 0$.

Remark 4. The reader may wonder whether the conditions in Theorem 1 and Corollary 1 preclude feasible tax systems, that is, tax systems that balance the government's budget. While no explicit feasibility conditions have been imposed on τ , it might be that inequality reducing tax systems are, after all, inconsistent with a balanced budget, which would invalidate the practical applicability of the results.

A progressive commodity tax system (Definition 4) taxes luxuries and (weakly) subsidizes necessities. Any progressive commodity tax system that uses the revenue from taxing luxuries to subsidize some (or all) necessities is *feasible*. Such tax systems are covered by Corollary 1 (and Theorem 1), provided that $\partial z^u(y, \mathbf{p}, \tau)/\partial y \geq 0$ for all $y > 0$

(which will hold, e.g., if the conditions in Remark 3 are fulfilled). However, because the agents' optimal consumption baskets depend not only on \mathbf{p} and τ , but also on the *initial* income distribution (y_1, \dots, y_n) , a feasible commodity tax system will, in general, vary with the pre-tax income distribution. Formally, given $u \in \mathcal{U}$, $\mathbf{p} \gg 0$, and for a *fixed* pre-tax income distribution (y_1, \dots, y_n) , a commodity tax system τ is feasible if

$$\sum_{i=1}^n (y_i - z^u(y_i, \mathbf{p}, \tau)) \geq 0$$

(recall (3)). Any tax system $\bar{\tau}$ with $\bar{\tau}_k > 0$ whenever k is a luxury and $\bar{\tau}_k = 0$ otherwise is progressive and satisfies

$$\sum_{i=1}^n (y_i - z^u(y_i, \mathbf{p}, \bar{\tau})) > 0$$

(provided, of course, that there is at least one luxury good). Now let $\underline{\tau}$ be a tax system such that $\underline{\tau}_k = \bar{\tau}_k$ whenever k is a luxury, $-p_k < \underline{\tau}_k < 0$ whenever k is a necessity, and $\underline{\tau} = 0$ otherwise. If

$$\sum_{i=1}^n (y_i - z^u(y_i, \mathbf{p}, \underline{\tau})) \geq 0$$

for any such $\underline{\tau}$, then *any* progressive tax system is feasible. If, alternatively, there exists one such $\underline{\tau}$ for which

$$\sum_{i=1}^n (y_i - z^u(y_i, \mathbf{p}, \underline{\tau})) < 0,$$

then, by virtue of the Intermediate Value Theorem, there is an $\alpha^* \in (0, 1)$ such that

$$\sum_{i=1}^n (y_i - z^u(y_i, \mathbf{p}, \alpha^* \bar{\tau} + (1 - \alpha^*) \underline{\tau})) = 0.$$

In this case, $\alpha \bar{\tau} + (1 - \alpha) \underline{\tau}$ is feasible for all $\alpha \in [\alpha^*, 1]$.

Remark 5. Consider a commodity tax system τ such that $\tau_k = \tau p_k$ for each k , for some $\tau \geq 0$. It is easy to see that τ is equivalent to a tax on commodity expenditures that levies $\$ \tau$ per dollar spent on any commodity. It is well known that such a proportional income tax is *neutral*, that is, it neither reduces nor increases inequality.

3 | ENDOGENOUS INCOME

The model in Section 2 is now augmented to allow for potential disincentive effects of taxation on work effort. To this end, the setting of Mirrlees (1971) is adapted to the present multigood context.

The social utility function is now given by $u: \mathbb{R}_+^K \times [0, 1] \rightarrow \mathbb{R}$ and defined over consumption-labor bundles (\mathbf{x}, l) , where $\mathbf{x} = (x_1, \dots, x_K)$ is a bundle of the $K \geq 1$ traded goods, and l measures labor supply. It is assumed that $u(\mathbf{x}, l)$ is strictly increasing in \mathbf{x} and strictly decreasing in l . It is also assumed that u is continuous and strictly quasi-concave, and that there exists $l > 0$ for which $u(\mathbf{x}, l) > u(\mathbf{0})$ whenever $\mathbf{x} \neq \mathbf{0}$.⁶ The last condition ensures that zero consumption is never optimal for an agent.

Let \mathcal{V} denote the collection of all utility functions satisfying the above conditions.

Agents differ in their abilities. An agent of ability $a > 0$ who chooses to supply $l \in [0, 1]$ units of labor earns income al , which can then be spent on any of the K commodities. Given a commodity price vector $\mathbf{p} = (p_1, \dots, p_K) \gg 0$ and a commodity tax system $\boldsymbol{\tau}$ with $\mathbf{p} + \boldsymbol{\tau} \gg 0$, the agent's problem is given by

$$\begin{aligned} \max_{(\mathbf{x}, l) \in \mathbb{R}_+^K \times [0, 1]} & u(\mathbf{x}, l), \\ \text{s.t.,} & \\ (\mathbf{p} + \boldsymbol{\tau})\mathbf{x} := & (p_1 + \tau_1)x_1 + \dots + (p_K + \tau_K)x_K \leq al. \end{aligned} \tag{8}$$

Because u is continuous and the feasible set is compact, this problem has at least one solution for each $(a, \mathbf{p}, \boldsymbol{\tau})$. Because u is strictly quasi-concave and the feasible set is convex, this solution is unique for each $(a, \mathbf{p}, \boldsymbol{\tau})$.

As pointed out by Christiansen (1984), the optimization problem in (8) can be solved in two stages. First, labor supply, l , is treated as fixed, and u is maximized with respect to \mathbf{x} . Once optimal commodity demands have been determined, conditional on l , the associated indirect utility function can be maximized with respect to l .

Let $l^u(a, \mathbf{p}, \boldsymbol{\tau})$ be the optimal labor supply at $(a, \mathbf{p}, \boldsymbol{\tau})$. The corresponding *pre-tax* and *post-tax income functions* are denoted by $y^u(a, \mathbf{p}, \boldsymbol{\tau})$ and $z^u(a, \mathbf{p}, \boldsymbol{\tau})$, respectively, and are given by

$$y^u(a, \mathbf{p}, \boldsymbol{\tau}) := al^u(a, \mathbf{p}, \boldsymbol{\tau})$$

and

$$\begin{aligned} z^u(a, \mathbf{p}, \boldsymbol{\tau}) &:= al^u(a, \mathbf{p}, \boldsymbol{\tau}) - \tau_1 X_1^u(\mathbf{p} + \boldsymbol{\tau}, y^u(a, \mathbf{p}, \boldsymbol{\tau})) - \dots - \tau_K X_K^u(\mathbf{p} + \boldsymbol{\tau}, y^u(a, \mathbf{p}, \boldsymbol{\tau})) \\ &= p_1 X_1^u(\mathbf{p} + \boldsymbol{\tau}, y^u(a, \mathbf{p}, \boldsymbol{\tau})) + \dots + p_K X_K^u(\mathbf{p} + \boldsymbol{\tau}, y^u(a, \mathbf{p}, \boldsymbol{\tau})) \end{aligned}$$

(recall that $X_1^u(\mathbf{p}, y), \dots, X_K^u(\mathbf{p}, y)$ represent the Marshallian demand functions for the K traded commodities).⁷ Note that the assumption that there exists $l > 0$ for which $u(\mathbf{x}, l) > u(\mathbf{0})$ whenever $\mathbf{x} \neq \mathbf{0}$ guarantees that $y^u(a, \mathbf{p}, \boldsymbol{\tau}) > 0$ for all $a > 0$.

Let \mathcal{V} denote the collection of utility functions $u \in \mathcal{V}$ such that for any $\mathbf{p} \gg 0$, the map $y \mapsto X_k^u(\mathbf{p}, y), k \in \{1, \dots, K\}$, is differentiable on \mathbb{R}_{++} , and the map $a \mapsto l^u(a, \mathbf{p}, \mathbf{0})$ is differentiable on \mathbb{R}_{++} .⁸ Given $u \in \mathcal{V}$, we have

⁶Our results hold intact if one considers instead quasi-linear or Cobb–Douglas utility functions, which fail strong monotonicity and strict quasi-concavity on (some of) the axes.

⁷Note that $l^u(a, \mathbf{p}, \boldsymbol{\tau}) = l^u(a, \mathbf{p} + \boldsymbol{\tau}, \mathbf{0})$, and so $y^u(a, \mathbf{p}, \boldsymbol{\tau}) = y^u(a, \mathbf{p} + \boldsymbol{\tau}, \mathbf{0})$.

⁸See Footnote 4. Here it suffices to require that these maps be right-differentiable on their domains. Under this weaker assumption, the partial derivatives in the statement of Theorem 2 should be replaced by their corresponding right partial derivatives.

$$z^u(a, \mathbf{p}, \tau) = y^u(a, \mathbf{p}, \tau) - \sum_{k=1}^K \tau_k X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau)) = \sum_{k=1}^K p_k X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau)) \quad (9)$$

and

$$\frac{\partial z^u(a, \mathbf{p}, \tau)}{\partial a} = \frac{\partial y^u(a, \mathbf{p}, \tau)}{\partial a} \left(1 - \sum_{k=1}^K \tau_k \cdot \frac{\partial X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))}{\partial y} \right) \quad (10)$$

$$= \frac{\partial y^u(a, \mathbf{p}, \tau)}{\partial a} \left(\sum_{k=1}^K p_k \cdot \frac{\partial X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))}{\partial y} \right). \quad (11)$$

Throughout the sequel, the following assumption will be maintained: The social utility function u induces a nondecreasing map $a \mapsto y^u(a, \mathbf{p}, \mathbf{0})$ defined on \mathbb{R}_{++} for every $\mathbf{p} \gg 0$. This condition is analogous to the standard *agent monotonicity* condition introduced by Mirrlees (1971, p. 182) (and named by Seade, 1982) in the one-good setting. It can be described intuitively as follows. An increase in the productivity parameter a triggers a substitution effect and an income effect on the hours worked. The substitution effect leads to a reduction in leisure time, which is now relatively more expensive. The income effect tends to reduce the number of hours worked (provided, of course, that leisure is a normal good). The above condition can be stated as follows: The substitution effect of an increase in a on l is not outweighed by the income effect.

Let \mathcal{V}^* be the set of all utility functions u in \mathcal{V} for which the map $a \mapsto y^u(a, \mathbf{p}, \mathbf{0})$ defined on \mathbb{R}_{++} is nondecreasing for every $\mathbf{p} \gg 0$.

For a fixed (but otherwise arbitrary) population size n , an *ability distribution* is a vector of abilities arranged in increasing order, that is, a vector $\mathbf{a} = (a_1, \dots, a_n) \gg 0$ with $a_1 \leq \dots \leq a_n$. Together with a price vector \mathbf{p} and a commodity tax system τ with $\mathbf{p} + \tau \gg 0$, an ability distribution $\mathbf{a} = (a_1, \dots, a_n)$ determines a *pre-tax income distribution*

$$\mathbf{y}^u(\mathbf{a}, \mathbf{p}, \tau) := (y^u(a_1, \mathbf{p}, \tau), \dots, y^u(a_n, \mathbf{p}, \tau))$$

and a *post-tax income distribution*

$$\mathbf{z}^u(\mathbf{a}, \mathbf{p}, \tau) := (z^u(a_1, \mathbf{p}, \tau), \dots, z^u(a_n, \mathbf{p}, \tau)).$$

When income is endogenous, the notion of income inequality reducing tax system is analogous to Definition 2.

Definition 5. Given $u \in \mathcal{V}^*$ and a price vector $\mathbf{p} \gg 0$, a commodity tax system τ with $\mathbf{p} + \tau \gg 0$ is *income inequality reducing with respect to* (u, \mathbf{p}) ((u, \mathbf{p}) -iir) if $\mathbf{z}^u(\mathbf{a}, \mathbf{p}, \tau) \succcurlyeq_L \mathbf{y}^u(\mathbf{a}, \mathbf{p}, \mathbf{0})$ for each ability distribution $\mathbf{a} \gg 0$.

Given $u \in \mathcal{V}^*$ and (\mathbf{p}, τ) , define

$$\xi^u(a, \mathbf{p}, \tau) := \frac{\partial y^u(a, \mathbf{p}, \tau)}{\partial a} \cdot \frac{a}{y^u(a, \mathbf{p}, \tau)};$$

$\xi^u(\cdot, \mathbf{p}, \tau)$ represents the wage elasticity of (before-tax) income given prices \mathbf{p} and tax system τ .

Remark 6. Define

$$\epsilon^u(a, \mathbf{p}, \tau) := \frac{\partial l^u(a, \mathbf{p}, \tau)}{\partial a} \cdot \frac{a}{l^u(a, \mathbf{p}, \tau)};$$

$\epsilon^u(\cdot, \mathbf{p}, \tau)$ is the wage elasticity of labor supply given prices \mathbf{p} and tax system τ . Since

$$\frac{\partial y^u(a, \mathbf{p}, \tau)}{\partial a} = a \cdot \frac{\partial l^u(a, \mathbf{p}, \tau)}{\partial a} + l^u(a, \mathbf{p}, \tau),$$

it follows that

$$\xi^u(a, \mathbf{p}, \tau) = 1 + \epsilon^u(a, \mathbf{p}, \tau).$$

The next result characterizes inequality reducing commodity tax systems. It extends Theorem 1 to the case of endogenous income.

Theorem 2. Given $u \in \mathcal{V}^*$ and a price vector $\mathbf{p} \gg 0$, a commodity tax system τ with $\mathbf{p} + \tau \gg 0$ is (u, \mathbf{p}) -iir if and only if $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$ and

$$\begin{aligned} & \xi^u(a, \mathbf{p}, \tau) \left(1 - \sum_{k=1}^K \tau_k \cdot \frac{\partial X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))}{\partial y} \right) \\ & \leq \xi^u(a, \mathbf{p}, \mathbf{0}) \left(1 - \sum_{k=1}^K \tau_k \cdot \frac{X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))}{y^u(a, \mathbf{p}, \tau)} \right) \end{aligned} \tag{12}$$

for all $a > 0$.

Remark 7. The condition that $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$ in Theorem 2 is analogous to the monotonicity condition on z^u from Theorem 1. Note that, in light of (10), we see that this condition holds if, for fixed \mathbf{p} , the derivatives $\partial X_k^u(\mathbf{p}', y)/\partial y$ ($k \in \{1, \dots, K\}$) are uniformly bounded over the pairs (\mathbf{p}', y) , where \mathbf{p}' represents a slight perturbation of \mathbf{p} , and where $y > 0$, and τ is sufficiently close to zero.

The proof of Theorem 2 is presented in the appendix.

Recall that a commodity is a luxury if and only if

$$\frac{\partial X_k^u(\mathbf{p}, y)}{\partial y} > \frac{X_k^u(\mathbf{p}, y)}{y} \quad \text{for each } (\mathbf{p}, y),$$

and it is a necessity if the same condition holds with the inequality reversed (Remark 2). Intuitively, the marginal effect of income on expenditure for the luxury commodity exceeds the proportion of

total income spent on the same commodity, so that the said proportion rises with income. Inspection of Condition (12) in the statement of Theorem 2 reveals that the condition defining luxuries (resp., necessities) has a direct bearing on the inequality reducing properties of a commodity tax system. Specifically, taxation of luxuries (resp., necessities) leads to a lower (resp., higher) bracketed term on the left-hand side of (12), relative to the bracketed term on the right-hand side. However, Condition (12) also indicates that the wage elasticity of income (or, as per Remark 6, the wage elasticity of labor supply) plays a role, too, in determining the effect of a tax system on income inequality. In particular, the effect of τ on ξ^u , which will generally differ across commodities, may work as a countervailing force.

Additional insight can be gained into Condition (12) by noting that a post-tax income distribution is more equally distributed than a pre-tax income distribution, regardless of the distribution of abilities, and according to the relative Lorenz ordering, if and only if the ratio of post-tax income to nontaxed income is nonincreasing in the ability parameter a (this is the content of Lemma 2 in Appendix B). Since we want the map $a \mapsto (z^u(a, \mathbf{p}, \tau)/y^u(a, \mathbf{p}, \mathbf{0}))$ to be nonincreasing, the ratio of the marginal effects must be less than the ratio of levels:

$$\frac{\partial z^u(a, \mathbf{p}, \tau)}{\partial a} \bigg/ \frac{\partial y^u(a, \mathbf{p}, \mathbf{0})}{\partial a} \leq \frac{z^u(a, \mathbf{p}, \tau)}{y^u(a, \mathbf{p}, \mathbf{0})} \quad \text{for all } a > 0.$$

In other words, the wage elasticity of disposable income, call it ζ^u , must be less than the wage elasticity of nontaxed income:

$$\zeta^u(a, \mathbf{p}, \tau) \leq \xi^u(a, \mathbf{p}, \mathbf{0}) \quad \text{for all } a > 0. \quad (13)$$

The rationale behind taxing luxury commodities more heavily can be understood as follows. Since $\zeta^u(a, \mathbf{p}, \tau)$ is expressible, via (9) and (10), as

$$\zeta^u(a, \mathbf{p}, \tau) = \xi^u(a, \mathbf{p}, \tau) \cdot \frac{1 - \sum_{k=1}^K \tau_k \cdot (\partial X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))/\partial y)}{1 - \sum_{k=1}^K \tau_k \cdot (X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))/y^u(a, \mathbf{p}, \tau))}, \quad (14)$$

we see that taxing luxury commodities helps reduce the wage elasticity of disposable income, ζ^u , relaxing the constraint in (13) via the ratio on the right-hand side of (14). Intuitively, because the marginal propensity to consume luxuries increases disproportionately with income, taxes on luxuries tend to reduce the sensitivity of disposable income with respect to ability more than proportionally. A similar argument can be made for the subsidization of necessities. Note, however, that τ has also a direct effect on $\zeta^u(a, \mathbf{p}, \tau)$, via the term $\xi^u(a, \mathbf{p}, \tau)$ in (14), that is, by changing the wage elasticity of pre-tax income. The sign of this effect is, in general, ambiguous, and, when positive (i.e., when $\xi^u(a, \mathbf{p}, \tau) > \xi^u(a, \mathbf{p}, \mathbf{0})$), it acts as a countervailing force. This direct effect did not appear in Section 2, where the income distribution was fixed. The case of exogenous income may be thought of as the case when the pre-tax income distribution is perfectly correlated with the ability distribution, implying that $\xi^u(a, \mathbf{p}, \tau) = \xi^u(a, \mathbf{p}, \mathbf{0}) = 1$, and so (13) reduces to

$$\frac{1 - \sum_{k=1}^K \tau_k \cdot (\partial X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))/\partial y)}{1 - \sum_{k=1}^K \tau_k \cdot (X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))/y^u(a, \mathbf{p}, \tau))} \leq 1,$$

which is precisely Condition (5) from Theorem 1.⁹

From (12), it is clear that if $\xi^u(a, \cdot, \mathbf{0})$ is nonincreasing in \mathbf{p} for each $a > 0$, then any progressive commodity tax system (recall Definition 4) τ that taxes luxuries but does not subsidize necessities, and satisfies $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$, is inequality reducing.

Corollary 2 (To Theorem 2). *Suppose that $u \in \mathcal{V}^*$ and there is a subset of luxury commodities, $\mathcal{K} \subseteq \{1, \dots, K\}$, such that, for each $k \in \mathcal{K}$, the map*

$$p_k \mapsto \xi^u(a, (p_1, \dots, p_{k-1}, p_k, p_{k+1}, \dots, p_K), \mathbf{0})$$

defined on \mathbb{R}_{++} is nonincreasing for each $a > 0$ and $(p_1, \dots, p_{k-1}, p_{k+1}, \dots, p_K) \gg 0$. Then, given $\mathbf{p} \gg 0$, a (progressive) commodity tax system τ with $\mathbf{p} + \tau \gg 0$, $\tau_k > 0$ for each $k \in \mathcal{K}$, and $\tau_k = 0$ for each $k \notin \mathcal{K}$ is (u, \mathbf{p}) -iir if and only if $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$.

If there is a subset of luxuries, \mathcal{K} , for which ξ^u is nonincreasing in its corresponding subset of prices, and there is a subset of necessities, \mathcal{K}^* , for which ξ^u is nondecreasing in the corresponding subset of prices, it follows from Theorem 2 that any progressive commodity tax system τ that taxes the goods in \mathcal{K} and subsidizes the goods in \mathcal{K}^* , and satisfies $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$, is inequality reducing.

Corollary 3 (To Theorem 2). *Suppose that $u \in \mathcal{V}^*$ and there is a subset of luxury commodities, $\mathcal{K} \subseteq \{1, \dots, K\}$, such that, for each $k \in \mathcal{K}$, the map*

$$p_k \mapsto \xi^u(a, (p_1, \dots, p_{k-1}, p_k, p_{k+1}, \dots, p_K), \mathbf{0})$$

defined on \mathbb{R}_{++} is nonincreasing for each $a > 0$ and $(p_1, \dots, p_{k-1}, p_{k+1}, \dots, p_K) \gg 0$. Suppose further that there is a subset of necessity commodities $\mathcal{K}^ \subseteq \{1, \dots, K\}$ such that, for each $k \in \mathcal{K}^*$, the map*

$$p_k \mapsto \xi^u(a, (p_1, \dots, p_{k-1}, p_k, p_{k+1}, \dots, p_K), \mathbf{0})$$

defined on \mathbb{R}_{++} is nondecreasing for each $a > 0$ and $(p_1, \dots, p_{k-1}, p_{k+1}, \dots, p_K) \gg 0$. Then, given $\mathbf{p} \gg 0$, a (progressive) commodity tax system τ with $\mathbf{p} + \tau \gg 0$, $\tau_k > 0$ for each $k \in \mathcal{K}$, $\tau_k \leq 0$ for each $k \in \mathcal{K}^$, and $\tau_k = 0$ elsewhere is (u, \mathbf{p}) -iir if and only if $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$.*

Remark 8. With enough differentiability, the monotonicity condition on the map $\mathbf{p} \mapsto \xi^u(a, \mathbf{p}, \mathbf{0})$ from Corollary 3 can be expressed as follows:

⁹Put differently, the exogenous income setting can be viewed, in the Mirrlees framework, as the particular case of costless work effort, so that each a -agent maximizes the number of hours worked (i.e., sets $l = 1$) and earns income a .

$$\frac{\partial \xi^u(a, \mathbf{p}, \mathbf{0})}{\partial p_k} = \frac{a}{y^u(a, \mathbf{p}, \mathbf{0})} \left(\frac{\partial^2 y^u(a, \mathbf{p}, \mathbf{0})}{\partial p_k \partial a} - \frac{1}{a} \cdot \xi^u(a, \mathbf{p}, \mathbf{0}) \cdot \frac{\partial y^u(a, \mathbf{p}, \mathbf{0})}{\partial p_k} \right) \leq 0 \text{ for each } a > 0, \mathbf{p} \gg 0, \text{ and } k \in \mathcal{K}.$$

A similar condition can be obtained for the monotonicity condition in Corollary 2.

Regarding the feasibility of the tax systems in Corollary 3, one can argue along the lines of Remark 4: For a *fixed* ability distribution, and starting from a feasible, progressive tax system like the ones considered in Corollary 2, one can introduce subsidies on necessities, in an incremental fashion, until the budget is balanced.

In the remainder of this section, we discuss a number of applications of Theorem 2.

3.1 | Uniform tax systems

Recall that a commodity tax system τ such that $\tau_k = \tau p_k$ for each k and some $\tau \geq 0$ is equivalent to a tax on commodity expenditures that levies τ per dollar spent on any commodity. A commodity tax system with these properties will be called a *uniform tax system*.

The next result states that for uniform tax systems the characterization given in Theorem 2 reduces to a monotonicity condition on the elasticity ξ^u .

Corollary 4 (To Theorem 2). *Given $u \in \mathcal{V}^*$ and a price vector $\mathbf{p} \gg 0$, a uniform commodity tax system τ is (u, \mathbf{p}) -iir if and only if*

$$\xi^u(a, \mathbf{p}, \tau) = \xi^u(a, \mathbf{p} + \tau, \mathbf{0}) \leq \xi^u(a, \mathbf{p}, \mathbf{0}) \text{ for all } a > 0,$$

which holds if and only if

$$\xi^u\left(\frac{a}{1 + \tau}, \mathbf{p}, \mathbf{0}\right) \leq \xi^u(a, \mathbf{p}, \mathbf{0}) \text{ for all } a > 0.$$

Proof. Fix $u \in \mathcal{V}^*$ and a price vector $\mathbf{p} \gg 0$. By Theorem 2, a uniform commodity tax system τ is (u, \mathbf{p}) -iir if and only if $\partial z^u(a, \mathbf{p}, \tau) / \partial a \geq 0$ for all $a > 0$ and (12) holds. The uniformity of τ gives, for each $a > 0$ (recall (9)),

$$\begin{aligned} z^u(a, \mathbf{p}, \tau) &= y^u(a, \mathbf{p}, \tau) - \sum_{k=1}^K \tau_k X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau)) \\ &= y^u(a, \mathbf{p}, \tau) - \tau \left(\sum_{k=1}^K p_k X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau)) \right) \\ &= y^u(a, \mathbf{p}, \tau) - \tau \cdot z^u(a, \mathbf{p}, \tau), \end{aligned}$$

implying that

$$z^u(a, \mathbf{p}, \tau) = \frac{y^u(a, \mathbf{p}, \tau)}{1 + \tau}, \tag{15}$$

and so $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ (since $y^u(\cdot, \mathbf{p}, \tau)$ is nondecreasing).

Now since (15) holds for all $a > 0$, it follows that

$$\frac{\partial z^u(a, \mathbf{p}, \tau)}{\partial a} = \frac{1}{1 + \tau} \cdot \frac{\partial y^u(a, \mathbf{p}, \tau)}{\partial a}.$$

This and (10) imply that

$$1 - \sum_{k=1}^K \tau_k \cdot \frac{\partial X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))}{\partial y} = \frac{1}{1 + \tau}.$$

In addition,

$$\begin{aligned} 1 - \sum_{k=1}^K \tau_k \cdot \frac{X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))}{y^u(a, \mathbf{p}, \tau)} &= 1 - \frac{\tau}{y^u(a, \mathbf{p}, \tau)} \left(\sum_{k=1}^K p_k X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau)) \right) \\ &= 1 - \tau \cdot \frac{z^u(a, \mathbf{p}, \tau)}{y^u(a, \mathbf{p}, \tau)} \\ &= 1 - \frac{\tau}{1 + \tau} \\ &= \frac{1}{1 + \tau}. \end{aligned}$$

Consequently, (12) reduces to

$$\xi^u(a, \mathbf{p}, \tau) \leq \xi^u(a, \mathbf{p}, \mathbf{0}) \text{ for all } a > 0. \tag{16}$$

Now since $l^u(a', \mathbf{p}, \tau) = l^u((a'/(1 + \tau)), \mathbf{p}, \mathbf{0})$ for all $a' > 0$,

$$\begin{aligned} y^u(a', \mathbf{p}, \tau) &= a' l^u(a', \mathbf{p}, \tau) = (1 + \tau) \left(\frac{a'}{1 + \tau} \right) l^u \left(\frac{a'}{1 + \tau}, \mathbf{p}, \mathbf{0} \right) \\ &= (1 + \tau) y^u \left(\frac{a'}{1 + \tau}, \mathbf{p}, \mathbf{0} \right) \end{aligned}$$

for all $a' > 0$. Consequently, for each $a' > 0$,

$$\frac{\partial y^u(a', \mathbf{p}, \tau)}{\partial a} = \frac{\partial y^u((a'/(1 + \tau)), \mathbf{p}, \mathbf{0})}{\partial a}$$

and

$$\begin{aligned}\xi^u(a', \mathbf{p}, \tau) &= \frac{\partial y^u(a', \mathbf{p}, \tau)}{\partial a} \cdot \frac{a'}{y^u(a', \mathbf{p}, \tau)} = \frac{\partial y^u((a'/(1+\tau)), \mathbf{p}, \mathbf{0})}{\partial a} \cdot \frac{(a'/(1+\tau))}{y^u((a'/(1+\tau)), \mathbf{p}, \mathbf{0})} \\ &= \xi^u\left(\frac{a'}{1+\tau}, \mathbf{p}, \mathbf{0}\right),\end{aligned}$$

implying that (16) is expressible as

$$\xi^u\left(\frac{a}{1+\tau}, \mathbf{p}, \mathbf{0}\right) \leq \xi^u(a, \mathbf{p}, \mathbf{0}) \quad \text{for all } a > 0,$$

as desired. □

3.2 | Commodities that are neither luxuries nor necessities

We now consider commodities that are neither luxuries nor necessities. These are commodities k whose underlying demand functions satisfy

$$\frac{\partial(p_k X_k^u(\mathbf{p}, y)/y)}{\partial y} = 0 \quad \text{for each } (\mathbf{p}, y).$$

This condition can be equivalently expressed as

$$\frac{\partial X_k^u(\mathbf{p}, y)}{\partial y} = \frac{X_k^u(\mathbf{p}, y)}{y} \quad \text{for each } (\mathbf{p}, y). \quad (17)$$

Homothetic preferences (including the constant elasticity of substitution (CES) utility function considered in Example 1) have this property. From (17), it follows that (12) holds if and only if

$$\xi^u(a, \mathbf{p}, \tau) = \xi^u(a, \mathbf{p} + \tau, \mathbf{0}) \leq \xi^u(a, \mathbf{p}, \mathbf{0}) \quad \text{for all } a > 0.$$

These observations are collected in the following result:

Corollary 5 (To Theorem 2). *Suppose that $u \in \mathcal{V}^*$ and*

$$\frac{\partial(p_k X_k^u(\mathbf{p}, y)/y)}{\partial y} = 0 \quad \text{for each } k \text{ and } (\mathbf{p}, y).$$

Given $\mathbf{p} \gg 0$, a commodity tax system τ with $\mathbf{p} + \tau \gg 0$ is (u, \mathbf{p}) -iir if and only if $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$ and $\xi^u(a, \mathbf{p}, \tau) = \xi^u(a, \mathbf{p} + \tau, \mathbf{0}) \leq \xi^u(a, \mathbf{p}, \mathbf{0})$ for all $a > 0$.

The following example illustrates Corollary 5 for the class of CES utility functions.

Example 1 (constant elasticity of substitution (CES) social utility). Consider the well-known CES utility function

$$u(\mathbf{x}, l) = \left((1 - l)^\gamma + \sum_{k=1}^K \alpha_k x_k^\gamma \right)^{1/\gamma}, \tag{18}$$

where $\alpha_k > 0$ for each $k \in \{1, \dots, K\}$ and $\gamma \in (-\infty, 1) \setminus \{0\}$. For each k ,

$$X_k^u(\mathbf{p}, y) = \left(\frac{(p_k/\alpha_k)^{1/(\gamma-1)}}{\sum_{k'} p_{k'} (p_{k'}/\alpha_{k'})^{1/(\gamma-1)}} \right) y.$$

Consequently, (17) holds and so for given \mathbf{p} , a commodity tax system τ is (u, \mathbf{p}) -iir only if

$$\xi^u(a, \mathbf{p} + \tau, \mathbf{0}) \leq \xi^u(a, \mathbf{p}, \mathbf{0}) \quad \text{for all } a > 0 \tag{19}$$

(Corollary 5). Routine calculations give

$$\xi^u(a, \mathbf{p}, \mathbf{0}) = \frac{1}{a(1 - \gamma)} \cdot \frac{1 + (1 - \gamma)A(\mathbf{p})a^{\gamma/(1-\gamma)}}{1 + A(\mathbf{p})a^{\gamma/(1-\gamma)}},$$

where

$$A(\mathbf{p}) := \left(\sum_k \alpha_k \left(\frac{(p_k/\alpha_k)^{1/(\gamma-1)}}{\sum_{k'} p_{k'} (p_{k'}/\alpha_{k'})^{1/(\gamma-1)}} \right)^\gamma \right)^{1/(\gamma-1)},$$

and since

$$\frac{\partial \xi^u(a, \mathbf{p}, \mathbf{0})}{\partial p_k} = -\frac{\gamma}{a(1 - \gamma)} \cdot \frac{a^{\gamma/(1-\gamma)} \cdot (\partial A(\mathbf{p})/\partial p_k)}{(1 + A(\mathbf{p})a^{\gamma/(1-\gamma)})^2}$$

and $\partial A(\mathbf{p})/\partial p_k < 0$ whenever $\gamma \in (0, 1)$ and $\partial A(\mathbf{p})/\partial p_k > 0$ whenever $\gamma < 0$, it follows that $\partial \xi^u(a, \mathbf{p}, \mathbf{0})/\partial p_k > 0$, implying that Condition (19) is never fulfilled, and so, for the utility function in (18), no commodity tax system is inequality reducing. However, in the limit as γ tends to 0 (in which case (18) collapses to the standard Cobb–Douglas utility function), one has $\xi^u(a, \mathbf{p}, \mathbf{0}) = 1/a$ for all (a, \mathbf{p}) , implying that Condition (19) is always fulfilled.

3.3 | An example

We conclude with an example illustrating Theorem 2.

Suppose that

$$u(x_1, x_2, l) = \ln(x_1 + 2\sqrt{x_2}) + \ln(1 - l).$$

For this utility function,

$$X_2^u(\mathbf{p}, w) = \begin{cases} \left(\frac{p_1}{p_2}\right)^2 & \text{if } \frac{p_1^2}{p_2} \leq w, \\ \frac{w}{p_2} & \text{otherwise,} \end{cases} \quad \text{and} \quad X_1^u(\mathbf{p}, w) = \begin{cases} \frac{w}{p_1} - \frac{p_1}{p_2} & \text{if } \frac{p_1^2}{p_2} \leq w, \\ 0 & \text{otherwise.} \end{cases}$$

Commodity 1 is a luxury, for

$$\frac{\partial X_1^u(\mathbf{p}, w)}{\partial w} = \begin{cases} \frac{1}{p_1} > \frac{1}{p_1} - \frac{p_1}{p_2 w} = \frac{X_1^u(\mathbf{p}, w)}{w} & \text{if } \frac{p_1^2}{p_2} < w, \\ 0 \geq 0 = \frac{X_1^u(\mathbf{p}, w)}{w} & \text{if } \frac{p_1^2}{p_2} > w. \end{cases}$$

Since there are only two goods, Commodity 2 must be a necessity.

One has

$$y^u(a, \mathbf{p}, \mathbf{0}) = \begin{cases} \frac{a}{2} - \frac{p_1^2}{2p_2} & \text{if } \frac{a}{3} \geq \frac{p_1^2}{p_2}, \\ \frac{a}{3} & \text{if } \frac{a}{3} < \frac{p_1^2}{p_2}, \end{cases}$$

and

$$\xi^u(a, \mathbf{p}, \mathbf{0}) = \begin{cases} \frac{ap_2}{ap_2 - p_1^2} & \text{if } \frac{a}{3} > \frac{p_1^2}{p_2}, \\ 1 & \text{if } \frac{a}{3} < \frac{p_1^2}{p_2}. \end{cases}$$

Because both goods are normal, $z^u(\cdot, \mathbf{p}, \tau)$ is nondecreasing in a for each (\mathbf{p}, τ) (recall (11)).

Now given $\mathbf{p} \gg 0$ and $a > 0$, consider the effects of a progressive commodity tax system τ whose burden is concentrated on the luxury good, that is, $\tau_1 > 0 = \tau_2$. To evaluate the inequality in (12), three cases are considered.

Case 1 $a/3 \geq (p_1 + \tau_1)^2/p_2$. In this case (12) is given by

$$\begin{aligned} & \frac{ap_2}{ap_2 - (p_1 + \tau_1)^2} \left(1 - \tau_1 \cdot \frac{1}{p_1 + \tau_1} \right) \\ & \leq \frac{ap_2}{ap_2 - p_1^2} \left[1 - \tau_1 \left(\frac{1}{p_1 + \tau_1} - \frac{2(p_1 + \tau_1)}{ap_2 - (p_1 + \tau_1)^2} \right) \right], \end{aligned}$$

which simplifies to

$$0 \leq 2\tau_1^3 + 3p_1\tau_1^2,$$

a true statement.

Case 2 $((p_1 + \tau_1)^2/p_2) > (a/3) \geq (p_1^2/p_2)$. Now (12) becomes

$$1 \leq \frac{ap_2}{ap_2 - p_1^2},$$

which holds because the right-hand side is greater than one.

Case 3 (p_1^2/p_2) > $a/3$. In this case, (12) reduces to $1 \leq 1$.

We conclude that for any price vector, any progressive commodity tax system τ with $\tau_1 > 0$ and $\tau_2 = 0$ is inequality reducing.

4 | CONCLUDING REMARKS

As a first step, the analysis conducted here disregards the case of “mixed taxation,” which covers the interaction between direct and indirect taxation. In a seminal paper, Atkinson and Stiglitz (1976) demonstrated that, in the presence of (nonlinear) direct taxation, indirect taxes are superfluous for social welfare maximization (under separability of utility between labor and consumption). The problem considered in this paper, that is, that of characterizing inequality reducing tax systems, differs from the social planner problem of Atkinson and Stiglitz (1976), and so there is no guarantee that the Atkinson–Stiglitz principle will extend to the present setting. In fact, in light of the results in Carbonell-Nicolau and Llavador (2018, 2019a), Condition (12) in Theorem 2, which characterizes the inequality reducing principle in the Ramsey setting, suggests that the taxation of luxury commodities might add an extra degree of freedom, in the mixed taxation setting, thereby relaxing the elasticity constraints obtained in Carbonell-Nicolau and Llavador (2018, 2019a). Indeed, the conditions on social preferences, derived in Carbonell-Nicolau and Llavador (2018, 2019a), required for a nonlinear income tax schedule to be inequality reducing preclude important classes of preferences. While, for such preferences, the *direct* taxation tools at the analyst’s disposal do not suffice, the possibility of targeting specifically luxury commodities, for which the marginal propensity to consume increases disproportionately with income, opens up the possibility that, in the present framework, mixed taxation may prove a more powerful instrument, one that achieves a decline in income inequality for a wider class of social preferences. A careful analysis of the mixed taxation case is left for future research.

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APPENDIX A: PROOF OF THEOREM 1

The proof of Theorem 1 is based on the following lemma, which adapts a well-known result (see, e.g., Le Breton et al., 1996, Proposition 3.1) to the present setting.

Lemma 1. Given $u \in \mathcal{U}$ and a price vector $\mathbf{p} \gg 0$, a commodity tax system τ with $\mathbf{p} + \tau \gg 0$ is (u, \mathbf{p}) -iir if and only if $(\partial z^u(y, \mathbf{p}, \tau)/\partial y) \geq 0$ for all $y > 0$ and the map

$$y \mapsto \frac{z^u(y, \mathbf{p}, \tau)}{y}$$

defined on \mathbb{R}_{++} is nonincreasing.

We now recall Theorem 1 and prove it.

Theorem 1. Given $u \in \mathcal{U}$ and a price vector $\mathbf{p} \gg 0$, a commodity tax system τ with $\mathbf{p} + \tau \gg 0$ is (u, \mathbf{p}) -iir if and only if $(\partial z^u(y, \mathbf{p}, \tau)/\partial y) \geq 0$ for all $y > 0$ and

$$\sum_{k=1}^K \tau_k \cdot \frac{X_k^u(\mathbf{p} + \tau, y)}{y} \leq \sum_{k=1}^K \tau_k \cdot \frac{\partial X_k^u(\mathbf{p} + \tau, y)}{\partial y} \quad \text{for all } y > 0. \quad (\text{A1})$$

Proof. Fix $u \in \mathcal{U}$ and a price vector $\mathbf{p} \gg 0$. By Lemma 1, a commodity tax system τ is (u, \mathbf{p}) -iir if and only if $(\partial z^u(y, \mathbf{p}, \tau)/\partial y) \geq 0$ for all $y > 0$ and the map $y \mapsto z^u(y, \mathbf{p}, \tau)/y$ defined on \mathbb{R}_{++} is nonincreasing. Since this map is differentiable, τ is (u, \mathbf{p}) -iir if and only if $\partial z^u(y, \mathbf{p}, \tau)/\partial y \geq 0$ for all $y > 0$ and

$$\frac{\partial z^u(y, \mathbf{p}, \tau)}{\partial y} \cdot y \leq z^u(y, \mathbf{p}, \tau) \quad \text{for all } y > 0.$$

This condition is expressible as

$$\left(1 - \tau_1 \cdot \frac{\partial X_1^u(\mathbf{p} + \tau, y)}{\partial y} - \dots - \tau_K \cdot \frac{\partial X_K^u(\mathbf{p} + \tau, y)}{\partial y}\right)y \leq y - \tau_1 X_1^u(\mathbf{p} + \tau, y) - \dots - \tau_K X_K^u(\mathbf{p} + \tau, y) \quad \text{for all } y > 0.$$

Rearranging terms gives (A1). □

APPENDIX B: PROOF OF THEOREM 2

The following preparatory lemma is instrumental in the proof of Theorem 2.

Lemma 2. *Given $u \in \mathcal{V}^*$ and a price vector $\mathbf{p} \gg 0$, a commodity tax system τ with $\mathbf{p} + \tau \gg 0$ is (u, \mathbf{p}) -iir if and only if $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$ and the map*

$$a \mapsto \frac{z^u(a, \mathbf{p}, \tau)}{y^u(a, \mathbf{p}, \mathbf{0})} \tag{B1}$$

defined on \mathbb{R}_{++} is nonincreasing.

Proof. Fix $u \in \mathcal{V}^*$ and $\mathbf{p} \gg 0$. The “if” part of the statement follows from Proposition 3.4 of Le Breton et al. (1996). Conversely, suppose that a commodity tax system τ is (u, \mathbf{p}) -iir. If $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$, then the map in (B1) is nonincreasing by Proposition 3.4 in Le Breton et al. (1996). Thus, it suffices to show that $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$. Proceeding by contradiction, suppose that $z^u(a, \mathbf{p}, \tau) > z^u(a', \mathbf{p}, \tau)$ for some a and a' with $0 < a < a'$. Because $u \in \mathcal{V}^*$ and $a' > a$, we have $y^u(a', \mathbf{p}, \tau) \geq y^u(a, \mathbf{p}, \tau)$. And since n can be chosen large enough so that

$$\frac{y^u(a, \mathbf{p}, \tau)}{(n - 1)y^u(a, \mathbf{p}, \tau) + y^u(a', \mathbf{p}, \tau)} > \frac{z^u(a', \mathbf{p}, \tau)}{(n - 1)z^u(a, \mathbf{p}, \tau) + z^u(a', \mathbf{p}, \tau)},$$

it follows that

$$\left(\underbrace{z^u(a, \mathbf{p}, \tau), \dots, z^u(a, \mathbf{p}, \tau)}_{n-1 \text{ times}}, z^u(a', \mathbf{p}, \tau) \right) \not\leq_L \left(\underbrace{y^u(a, \mathbf{p}, \tau), \dots, y^u(a, \mathbf{p}, \tau)}_{n-1 \text{ times}}, y^u(a, \mathbf{p}, \tau) \right),$$

implying that τ is not (u, \mathbf{p}) -iir. □

We now recall Theorem 2 and prove it.

Theorem 2. *Given $u \in \mathcal{V}^*$ and a price vector $\mathbf{p} \gg 0$, a commodity tax system τ with $\mathbf{p} + \tau \gg 0$ is (u, \mathbf{p}) -iir if and only if $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$ and*



$$\begin{aligned} & \xi^u(a, \mathbf{p}, \tau) \left(1 - \sum_{k=1}^K \tau_k \cdot \frac{\partial X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))}{\partial y} \right) \\ & \leq \xi^u(a, \mathbf{p}, \mathbf{0}) \left(1 - \sum_{k=1}^K \tau_k \cdot \frac{X_k^u(\mathbf{p} + \tau, y^u(a, \mathbf{p}, \tau))}{y^u(a, \mathbf{p}, \tau)} \right) \text{ for all } a > 0. \end{aligned} \tag{B2}$$

Proof. Fix $u \in \mathcal{V}^*$ and a price vector $\mathbf{p} \gg 0$. By Lemma 2, a commodity tax system τ is (u, \mathbf{p}) -iir if and only if $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$ and the map $a \mapsto z^u(a, \mathbf{p}, \tau)/y^u(a, \mathbf{p}, \mathbf{0})$ defined on \mathbb{R}_{++} is nonincreasing. Since this map is differentiable, τ is (u, \mathbf{p}) -iir if and only if $\partial z^u(a, \mathbf{p}, \tau)/\partial a \geq 0$ for all $a > 0$ and

$$\frac{\partial z^u(a, \mathbf{p}, \tau)}{\partial a} \cdot y^u(a, \mathbf{p}, \mathbf{0}) \leq \frac{\partial y^u(a, \mathbf{p}, \mathbf{0})}{\partial a} \cdot z^u(a, \mathbf{p}, \tau) \text{ for all } a > 0.$$

Using (9) and (10) and arranging terms gives (B2). □