

On Dynamic Income Taxation and Absolute Inequality*

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Abstract

This paper evaluates income tax schedules by their ability to reduce absolute inequality in a dynamic setting. Classical static absolute-equalization criteria permit a broad class of nonlinear tax schedules, but they do not assess whether a tax system mitigates changes in inequality over time. We introduce Dynamic Absolute Equalization and show that a continuous tax schedule satisfies this criterion if and only if it is affine, with a marginal tax rate between zero and one and a nonpositive intercept. Dynamic equalization thus imposes a substantially sharper restriction than static absolute equalization.

Keywords: Absolute inequality, dynamic Lorenz dominance, progressive taxation, affine taxation.

JEL classifications: D63, D71, H23.

1 Introduction

We evaluate a tax schedule by its ability to unambiguously reduce income inequality. The seminal works of Jakobsson (1976), Fellman (1976), and Kakwani (1977) pioneered this methodology for relative inequality in endowment economies. Moyes (1987) introduced absolute Lorenz dominance as a criterion for comparing absolute inequality. Building on this criterion, Moyes (1988) showed that a tax schedule reduces absolute inequality for every

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pre-tax distribution—a property termed Uniform Absolute Equalization (UAE)—if and only if both post-tax income and tax liability are weakly increasing in pre-tax income.

While foundational, these characterizations are fundamentally static. They evaluate the equalizing power of a tax schedule at a single point in time. In reality, pre-tax distributions evolve over time, and macroeconomic shocks or technological shifts may exacerbate baseline inequality. A static framework cannot assess whether a tax schedule acts as an effective intertemporal stabilizer across such transitions. Evaluating this requires a dynamic normative framework, which we operationalize using the absolute dynamic Lorenz dominance criterion developed by [Carbonell-Nicolau \(2026\)](#).

In this note, we introduce two dynamic normative criteria. Dynamic Absolute Equalization (DAE) requires taxation to weakly mitigate any unambiguous intertemporal deterioration in absolute inequality. Responsive Absolute Equalization (RAE) requires taxation’s equalizing effect to be weakly stronger at the more unequal distribution.

We first establish that these two dynamic criteria are equivalent. We then present our main result ([Theorem 1](#)): a continuous tax schedule satisfies Dynamic Absolute Equalization if and only if it belongs to a restricted class of affine schedules. The intuition for the affine characterization is straightforward. While the static UAE criterion accommodates a broad class of nonlinear schedules, demanding intertemporal consistency across changing baseline inequalities forces the tax system to mitigate income gaps uniformly, regardless of where those gaps occur in the distribution. Because absolute dynamic Lorenz dominance evaluates translation-invariant income shortfalls, universal dynamic mitigation requires a constant marginal tax rate. Nevertheless, the resulting affine schedules are average-rate progressive and guarantee static absolute equalization.

2 Framework and definitions

2.1 Income distributions

The set of feasible income distributions is denoted by

$$\mathcal{D} = \bigcup_{n=1}^{\infty} \mathcal{D}_n,$$

where

$$\mathcal{D}_n = \left\{ \mathbf{z} = (z_1, \dots, z_n) \in \mathbb{R}_+^n : \sum_{i=1}^n z_i > 0 \right\}.$$

For any distribution $\mathbf{z} \in \mathcal{D}$, let $n_{\mathbf{z}}$ denote its population size and $\mu_{\mathbf{z}}$ its mean. Let

$$z_{[1]} \leq \dots \leq z_{[n_{\mathbf{z}}]}$$

denote its ordered incomes.

2.2 Tax schedules

A *tax schedule* is defined by a function $t : \mathbb{R}_+ \rightarrow \mathbb{R}$, which assigns a tax liability $t(z)$ to any pre-tax income $z \in \mathbb{R}_+$. The tax liability $t(z)$ represents either a tax (if $t(z) > 0$) or a subsidy (if $t(z) < 0$).

For any pre-tax distribution $\mathbf{z} \in \mathcal{D}$, define the vector of post-tax incomes by

$$\tilde{\mathbf{z}}^t = (z_{[1]} - t(z_{[1]}), \dots, z_{[n_{\mathbf{z}}]} - t(z_{[n_{\mathbf{z}}]})),$$

and let $\mathbf{z}^t$ denote its nondecreasing rearrangement.

2.3 Static and dynamic inequality

Following [Moyes \(1987\)](#), we evaluate absolute inequality using the absolute Lorenz curve. For any distribution $\mathbf{z} \in \mathcal{D}$, define its absolute Lorenz curve $AL(p; \mathbf{z})$ at the discrete population shares by

$$AL\left(\frac{k}{n_{\mathbf{z}}}; \mathbf{z}\right) = \frac{1}{n_{\mathbf{z}}} \sum_{j=1}^k (z_{[j]} - \mu_{\mathbf{z}}), \quad \text{for } k \in \{1, \dots, n_{\mathbf{z}}\}.$$

Set $AL(0; \mathbf{z}) = 0$ and extend the curve to every $p \in [0, 1]$ by linear interpolation. The negative of $AL(p; \mathbf{z})$ is the cumulative shortfall from mean income of the poorest p fraction, divided by the total population size.

Definition 1 (Absolute Lorenz Dominance, $\succ_{AL}$). For any $\mathbf{z}, \mathbf{z}' \in \mathcal{D}$, $\mathbf{z}'$ weakly dominates $\mathbf{z}$ in the absolute Lorenz sense, denoted $\mathbf{z}' \succ_{AL} \mathbf{z}$, if

$$AL(p; \mathbf{z}') \geq AL(p; \mathbf{z}), \quad \text{for every } p \in [0, 1].$$

To evaluate the evolution of inequality over time, we consider a transition $(\mathbf{x}, \mathbf{y})$ from an initial income distribution $\mathbf{x} \in \mathcal{D}$ to a final income distribution $\mathbf{y} \in \mathcal{D}$. We compare such transitions using the Absolute Dynamic Lorenz dominance criterion introduced and characterized axiomatically in [Carbonell-Nicolau \(2026\)](#).

Definition 2 (Absolute Dynamic Lorenz Dominance, $\succ_{ADL}$). For any $(\mathbf{x}', \mathbf{y}'), (\mathbf{x}, \mathbf{y}) \in \mathcal{D} \times \mathcal{D}$, $(\mathbf{x}', \mathbf{y}')$ weakly dominates $(\mathbf{x}, \mathbf{y})$ in the absolute dynamic Lorenz sense, denoted

$$(\mathbf{x}', \mathbf{y}') \succ_{ADL} (\mathbf{x}, \mathbf{y}),$$

if

$$AL(p; \mathbf{y}') - AL(p; \mathbf{x}') \geq AL(p; \mathbf{y}) - AL(p; \mathbf{x}), \quad \text{for every } p \in [0, 1].$$

2.4 Uniform Absolute Equalization

Following [Moyes \(1988\)](#), we define the property of Uniform Absolute Equalization (UAE) to characterize taxation schemes that unambiguously reduce absolute income inequality for any given baseline distribution.

Definition 3 (Uniform Absolute Equalization, UAE). A tax schedule t satisfies **UAE** if, for all pre-tax distributions $\mathbf{z} \in \mathcal{D}$, the corresponding post-tax distribution $\mathbf{z}^t$ weakly dominates $\mathbf{z}$ in the absolute Lorenz sense: $\mathbf{z}^t \succ_{AL} \mathbf{z}$.

Using this definition, the main characterization in [Moyes \(1988\)](#) states that a tax schedule satisfies **UAE** if and only if both the tax liability and post-tax income are weakly increasing in pre-tax income.

2.5 Normative criteria for dynamic inequality

We consider an economy that transitions from an initial pre-tax income distribution $\mathbf{x} \in \mathcal{D}$ to a final pre-tax distribution $\mathbf{y} \in \mathcal{D}$. Our criteria apply when absolute inequality weakly worsens at every population share. Formally, we restrict attention to transitions satisfying $\mathbf{x} \succ_{AL} \mathbf{y}$, so that

$$AL(p; \mathbf{y}) - AL(p; \mathbf{x}) \leq 0, \quad \text{for every } p \in [0, 1].$$

This restriction makes the criterion one-sided: it asks whether taxation mitigates transitions in which pre-tax inequality unambiguously worsens. Without it, the requirement would

apply to every transition (x, y) and also to its reverse (y, x) . The first application would place a lower bound on the post-tax change in each absolute Lorenz ordinate, while the reverse application would place the corresponding upper bound. The two bounds would force the post-tax and pre-tax changes to coincide at every population share. Dropping the Lorenz-order restriction would thus strengthen the criterion from mitigation of inequality increases to exact preservation of all inequality changes.

For transitions satisfying $x \succsim_{AL} y$, the first criterion requires taxation to make every deterioration in absolute inequality weakly less severe.

Definition 4 (Dynamic Absolute Equalization, DAE). A tax schedule t satisfies **DAE** if, for every pre-tax transition (x, y) such that $x \succsim_{AL} y$, the post-tax distributions x^t and y^t belong to $\mathcal{D}$ and

$$(x^t, y^t) \succsim_{ADL} (x, y).$$

Because $x \succsim_{AL} y$, **DAE** does not require the increase in pre-tax inequality to persist after taxation. It permits taxation to attenuate, eliminate, or reverse that increase.

The second criterion expresses the stabilization principle by comparing the equalizing effect of taxation across the two pre-tax distributions. It requires this effect to be weakly stronger at y , the more unequal distribution.

Definition 5 (Responsive Absolute Equalization, RAE). A tax schedule t satisfies **RAE** if, for every pre-tax transition (x, y) such that $x \succsim_{AL} y$, the post-tax distributions x^t and y^t belong to $\mathcal{D}$ and

$$(y, y^t) \succsim_{ADL} (x, x^t).$$

2.6 Characterization of DAE

We first establish the algebraic equivalence of our two dynamic normative criteria.

Lemma 1. **DAE is equivalent to RAE.**

Proof. Both **DAE** and **RAE** require $x^t, y^t \in \mathcal{D}$ for every pre-tax transition (x, y) satisfying $x \succsim_{AL} y$. It therefore remains to show that their respective dominance requirements are equivalent.

By **Definition 4**, **DAE** requires

$$(x^t, y^t) \succsim_{ADL} (x, y).$$

Applying **Definition 2**, this is equivalent to

$$AL(p; \mathbf{y}^t) - AL(p; \mathbf{x}^t) \geq AL(p; \mathbf{y}) - AL(p; \mathbf{x}) \quad \text{for every } p \in [0, 1].$$

Rearranging gives

$$AL(p; \mathbf{y}^t) - AL(p; \mathbf{y}) \geq AL(p; \mathbf{x}^t) - AL(p; \mathbf{x}) \quad \text{for every } p \in [0, 1].$$

By **Definition 2**, this inequality is equivalent to

$$(\mathbf{y}, \mathbf{y}^t) \succ_{ADL} (\mathbf{x}, \mathbf{x}^t),$$

which is the dominance requirement in **Definition 5**. Because this algebraic rearrangement is reversible and the two criteria impose the same post-tax feasibility requirement, **DAE** and **RAE** are equivalent. $\blacksquare$

Next, we establish the main characterization theorem. We show that **DAE** uniquely characterizes the class of affine tax schedules.

Theorem 1. *A continuous tax schedule $t : \mathbb{R}_+ \rightarrow \mathbb{R}$ satisfies **DAE** if and only if it takes the form*

$$t(z) = \alpha z + \beta,$$

for some $\alpha \in [0, 1]$ and $\beta \leq 0$, with $\beta < 0$ when $\alpha = 1$.

Proof. Sufficiency. Let $t(z) = \alpha z + \beta$ with $\alpha \in [0, 1]$ and β restricted as above. Because $1 - \alpha \geq 0$, the post-tax income mapping is weakly increasing and therefore preserves ranks. Hence, for any $\mathbf{z} \in \mathcal{D}$,

$$z_{[i]}^t = (1 - \alpha)z_{[i]} - \beta \quad \text{for every } i.$$

Since $z_{[i]} \geq 0$, $\alpha \leq 1$, and $\beta \leq 0$, we have $z_{[i]}^t \geq 0$ for every i . Moreover,

$$\sum_{i=1}^{n_z} z_{[i]}^t = (1 - \alpha) \sum_{i=1}^{n_z} z_{[i]} - n_z \beta > 0$$

because either $1 - \alpha > 0$ (and $\sum_i z_{[i]} > 0$), or $\alpha = 1$ and $\beta < 0$. Thus, $\mathbf{z}^t \in \mathcal{D}$.

Now consider any admissible pre-tax transition $(\mathbf{x}, \mathbf{y})$ such that $\mathbf{x} \succ_{AL} \mathbf{y}$. The common

intercept $-\beta$ cancels after mean-centering, so

$$AL(p; \mathbf{z}^t) = (1 - \alpha)AL(p; \mathbf{z}), \quad \text{for every } \mathbf{z} \in \mathcal{D} \text{ and } p \in [0, 1].$$

By definition, $AL(p; \mathbf{y}) - AL(p; \mathbf{x}) \leq 0$ for every $p \in [0, 1]$. Since $0 \leq 1 - \alpha \leq 1$,

$$(1 - \alpha)[AL(p; \mathbf{y}) - AL(p; \mathbf{x})] \geq AL(p; \mathbf{y}) - AL(p; \mathbf{x}).$$

Using the preceding identity gives

$$AL(p; \mathbf{y}^t) - AL(p; \mathbf{x}^t) \geq AL(p; \mathbf{y}) - AL(p; \mathbf{x}),$$

which establishes **DAE**.

Necessity. Assume that t is continuous and satisfies **DAE**. Because **DAE** must hold for every admissible transition, it must hold for the subset of transitions where $n_x = n_y = 2$. We denote the post-tax income mapping as $g(z) = z - t(z)$.

For any two-person distribution $\mathbf{z} = (z_1, z_2)$ with $z_1 \leq z_2$, the absolute Lorenz curve evaluated at $p = 1/2$ is

$$AL(1/2; \mathbf{z}) = -(z_2 - z_1)/4.$$

Hence

$$AL(1/2; \mathbf{z}^t) = -|g(z_2) - g(z_1)|/4.$$

The **DAE** condition

$$AL(1/2; \mathbf{y}^t) - AL(1/2; \mathbf{x}^t) \geq AL(1/2; \mathbf{y}) - AL(1/2; \mathbf{x})$$

can be written as

$$-|g(y_2) - g(y_1)| + |g(x_2) - g(x_1)| \geq -(y_2 - y_1) + (x_2 - x_1).$$

When the two pre-tax income gaps are equal, $x_2 - x_1 = y_2 - y_1 = h \geq 0$, the two distributions have identical absolute Lorenz curves. Hence both $\mathbf{x} \succ_{AL} \mathbf{y}$ and $\mathbf{y} \succ_{AL} \mathbf{x}$ hold. Applying **DAE** in both directions yields

$$|g(x_1 + h) - g(x_1)| = |g(y_1 + h) - g(y_1)|.$$

Thus, for every $h \geq 0$, the absolute increment $|g(a+h) - g(a)|$ is independent of the base point $a \geq 0$. Fix $h > 0$, and denote this base-point-independent absolute increment by $\phi(h)$. If $\phi(h) = 0$, then $g(a+h) - g(a) = 0$ for every $a \geq 0$. If $\phi(h) > 0$, the continuous function $a \mapsto g(a+h) - g(a)$ never vanishes. Because $\mathbb{R}_+$ is connected, this function has constant sign. Since its absolute value is also constant, there exists $\psi(h) \in \mathbb{R}$ such that $g(a+h) - g(a) = \psi(h)$ for every $a \geq 0$. Set $\psi(0) = 0$. It follows that $g(a+h) - g(a) = \psi(h)$ for all $a, h \geq 0$.

Setting $a = 0$ gives $g(h) = g(0) + \psi(h)$. Moreover, for all $h, k \geq 0$,

$$\psi(h+k) = g(a+h+k) - g(a) = [g(a+h+k) - g(a+h)] + [g(a+h) - g(a)] = \psi(k) + \psi(h).$$

Therefore, ψ is additive on $\mathbb{R}_+$. Since $g(h) = g(0) + \psi(h)$ and g is continuous, ψ is continuous. Additivity implies that $\psi(m) = m\psi(1)$ for every nonnegative integer m . For every positive integer n , additivity also gives

$$\psi(1) = n\psi\left(\frac{1}{n}\right),$$

so

$$\psi\left(\frac{1}{n}\right) = \frac{\psi(1)}{n}.$$

Hence,

$$\psi(q) = q\psi(1)$$

for every nonnegative rational number q . Because the nonnegative rational numbers are dense in $\mathbb{R}_+$ and ψ is continuous, the same identity holds for every $h \geq 0$. Thus, setting $c = \psi(1)$, we obtain $\psi(h) = ch$ for every $h \geq 0$. Consequently, $g(z) = g(0) + cz$. Writing $g(0) = -\beta$, we have $g(z) = cz - \beta$. Therefore,

$$t(z) = z - g(z) = (1 - c)z + \beta.$$

Letting $\alpha = 1 - c$, we obtain $t(z) = \alpha z + \beta$.

Substituting $g(z) = (1 - \alpha)z - \beta$ back into the **DAE** inequality for any two-person transition satisfying $y_2 - y_1 \geq x_2 - x_1$ yields

$$(1 - |1 - \alpha|) \left[(y_2 - y_1) - (x_2 - x_1) \right] \geq 0.$$

Because admissible transitions include pairs for which $(y_2 - y_1) - (x_2 - x_1) > 0$, this requires $1 - |1 - \alpha| \geq 0$, or equivalently $\alpha \in [0, 2]$.

Finally, **DAE** implies that $\mathbf{z}^t \in \mathcal{D}$ for every $\mathbf{z} \in \mathcal{D}$. Indeed, the stationary transition $(\mathbf{z}, \mathbf{z})$ satisfies $\mathbf{z} \succ_{AL} \mathbf{z}$, so the post-tax feasibility requirement in **Definition 4** applies. We now show that

$$g(z) \geq 0, \quad \text{for every } z \geq 0.$$

For $z > 0$, apply the preceding conclusion to the one-person distribution (z) . Because $\mathcal{D}$ requires strictly positive aggregate income, $(0) \notin \mathcal{D}$; the case $z = 0$ follows by applying the same conclusion to a feasible distribution containing zero, such as $(0, 1)$.

Since

$$g(z) = (1 - \alpha)z - \beta,$$

this is possible only if $1 - \alpha \geq 0$, for otherwise $g(z) \rightarrow -\infty$ as $z \rightarrow \infty$. Hence $\alpha \leq 1$. Combining this with $\alpha \in [0, 2]$ gives $\alpha \in [0, 1]$. Moreover, $g(0) = -\beta \geq 0$, so $\beta \leq 0$. If $\alpha = 1$, all post-tax incomes equal $-\beta$. The post-tax feasibility requirement in **DAE** then implies $\beta < 0$, since $\beta = 0$ would map every distribution to the zero vector, which does not belong to $\mathcal{D}$. ■

The characterization in **Theorem 1** has two further implications.

First, the characterized schedules satisfy the conditions established by **Moyes (1988)** for static absolute equalization. Under our characterization, the tax liability $t(z) = \alpha z + \beta$ is weakly increasing because $\alpha \geq 0$, while post-tax income $z - t(z) = (1 - \alpha)z - \beta$ is weakly increasing because $1 - \alpha \geq 0$. Thus, every schedule satisfying the dynamic criterion also satisfies static **UAE**.

Second, the characterization implies average-rate progressivity, which is not required by static absolute equalization. For $z > 0$, the average tax rate is

$$\frac{t(z)}{z} = \alpha + \frac{\beta}{z}.$$

Because $\beta \leq 0$, it is weakly increasing in pre-tax income.

3 Concluding remarks

This note extends the Lorenz-based evaluation of income taxation from a static to a dynamic setting. Requiring taxation to mitigate intertemporal increases in absolute inequality sharply

restricts the admissible class. While static absolute equalization accommodates nonlinear schedules, the dynamic criterion, including its post-tax feasibility requirement, confines the admissible class to affine schedules with the parameter restrictions stated in [Theorem 1](#).

A natural extension concerns dynamic relative inequality. [Carbonell-Nicolau \(2026\)](#) develops a relative dynamic Lorenz criterion based on centered log-income changes. This criterion is defined for strictly positive incomes. Under it, uniform proportional growth is neutral, and transitions are compared through cumulative changes in the centered log incomes of lower-ranked groups. An alternative criterion can be based on proportional changes in ordinary Lorenz ordinates.¹ Let $L(p; \mathbf{z})$ denote the income share received by the poorest p fraction of the population in distribution $\mathbf{z}$. For any pre-tax distributions $\mathbf{x}$ and $\mathbf{y}$ such that $\mathbf{x}$ weakly Lorenz dominates $\mathbf{y}$, the alternative criterion requires

$$L(p; \mathbf{y}^t) L(p; \mathbf{x}) \geq L(p; \mathbf{y}) L(p; \mathbf{x}^t), \quad \text{for every } p \in (0, 1).$$

When the relevant Lorenz ordinates are positive, this condition equivalently requires the post-tax transition to exhibit a weakly smaller proportional deterioration in lower-group income shares than the pre-tax transition, or the proportional equalizing effect of taxation to be weakly stronger at $\mathbf{y}$ than at $\mathbf{x}$. The cross-product formulation remains well defined when some Lorenz ordinates are zero. Thus, as in the absolute case, the analogues of [DAE](#) and [RAE](#) coincide under the alternative criterion. The same equivalence holds under the centered log-income criterion by the argument used in [Lemma 1](#). The alternative criterion differs from the centered log-income criterion by applying the multiplicative comparison directly to ordinary Lorenz shares. Characterizing the tax schedules that satisfy these relative criteria is left for future research.

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