Oriol Carbonell-Nicolau[1]

Unbundling Production with Decreasing Average Costs

[1] Department of Economics, Rutgers University, 75 Hamilton Street, New Brunswick, NJ 08901, USA, E-mail: carbonell-nicolau@rutgers.edu

Abstract:
This paper argues that, even in the presence of decreasing average costs of production, monopolies can sometimes be avoided in the interest of market efficiency. It is shown that under certain conditions on the variable cost of production, there is an alternate, viable market structure that reduces the well-known deadweight loss of monopoly: an "upstream" market in which one or more firms own and share the fixed cost of creating and maintaining a distribution network, and a "downstream" market populated by a large number of firms that are charged a unit price for the network's usage.

Keywords: natural monopoly, Cournot competition, deadweight loss of monopoly

JEL classification: L12, L13

DOI: 10.1515/bejte-2018-0097

1 Introduction

When average cost functions are decreasing, or when they are decreasing over a sufficiently wide range of output levels, so that the minimum of the average cost curve lies to the right of the demand curve, it is sometimes argued that markets are better served by a sole vendor. This situation is commonly known as the case of a *natural monopoly*. More generally, natural monopolies are sometimes defined by the existence of *strictly subadditive* cost functions, i. e. cost functions $C(q)$ satisfying, for each $q \geq 0$, $k \in \mathbb{N}$, and $(q_1, ..., q_k) \in \mathbb{R}^k_+$ with $q_1 + ... + q_k = q$,[1]

$$C(q) < C(q_1) + ... + C(q_k); \tag{1}$$

These cost structures imply that any given q can be produced more cheaply by one single firm than by any number k of firms.[2,3]

The case of public utilities—where the technology involves very large fixed costs and small marginal costs—is often given as the "perfect" example of a natural monopoly. In this case, the fixed costs are associated with creating and maintaining a distribution network—be it gas or water delivery pipes, wires and switching networks, etc. It is not "efficient," so the argument goes, for several water companies to lay the pipes for water delivery all over a given town.

This paper shows that a case against "natural monopolies" can be made, under some conditions. Specifically, it is shown that there is an alternate, viable market structure—with an "upstream" market in which one or more firms own and share the costs of creating and maintaining the distribution network, and a "downstream" market (i. e. the service market at the last stage of the production/distribution chain) populated by a large number of firms that are charged a unit price for the network's usage—that is more efficient than a monopoly.

The analysis conducted here bears a relationship to the literature on monopoly regulation and the literature on vertical (dis)integration, access pricing, and vertical foreclosure. This relationship is discussed below (see the discussion following Proposition 1 in Section 3).

The main point of the paper is illustrated, in simple terms, via an example, in Section 2. The general case is presented in Section 3.

2 An Example

Assuming a cost function $C(q) = F + \alpha q^2$, where q denotes output, $F > 0$ represents the fixed cost of creating and maintaining the distribution network, and where α is positive, yields a strictly decreasing average cost function

Oriol Carbonell-Nicolau is the corresponding author.

$AC(q) = \frac{F}{q} + \alpha q$ over the range $0 \le q < \sqrt{\frac{F}{\alpha}}$. Thus, given a linear market inverse demand function, $p(q) = a - bq$, where $a > 0$ and $b > 0$, the minimum of the average cost curve lies to the right of the inverse demand curve if $\frac{a}{b} < \sqrt{\frac{F}{\alpha}}$. In this case, one has a "natural monopoly," in the sense that average cost is decreasing over the relevant output range, $0 \le q \le \frac{a}{b}$. In addition, it is easy to verify that $C(q)$ is strictly subadditive (recall eq. (1)) over the relevant output range, $0 \le q \le \frac{a}{b}$, provided that $\frac{a}{b} < \sqrt{\frac{F}{\alpha}}$.

The monopoly output allocation is $q_{mon} \equiv \frac{a}{2(\alpha+b)}$, and the corresponding monopoly profit is

$$\pi_{mon} \equiv p(q_{mon})q_{mon} - C(q_{mon}) = \frac{a^2}{4(\alpha + b)} - F.$$

Suppose that $\pi_{mon} > 0$.

We wish to compare the monopoly allocation with that arising in a "segmented" market in which an upstream firm, call it M, owns the distribution network and charges firms in the downstream market for service delivery over the distribution network. As will be clear from the analysis leading to Proposition 1 (in Section 3 below), when the downstream market is populated by a large number of firms, the upstream monopolist serves approximately the output level $q_M \approx \frac{a}{2b} > q_{mon}$ units at price $p_M \approx \frac{a}{2}$, which yields a profit of approximately $\frac{a^2}{4b} - F > \pi_{mon} > 0$.

One can gain insight from the comparison of the two market structures. First, because $q_M > q_{mon}$, the segmented market yields a lower deadweight loss than a monopoly. Second, if a monopoly is viable (i. e. if $\pi_{mon} > 0$) then so is the upstream monopolist, whose profits are given by $\frac{a^2}{4b} - F > \pi_{mon} > 0$. In addition, as will be clear from the general analysis, the downstream firms also make positive profits. Third, because the upstream monopolist makes more profits than a sole vendor, the segmented market structure benefits not only the consumers, but also the monopolist. Intuitively, as the number of downstream firms increases, the markup in the downstream market decreases (and vanishes in the limit case of perfect downstream competition); in addition, if the marginal variable cost, $c'(\cdot)$, is increasing in q (as is the case in the above example), downstream competition reduces downstream production costs (relative to the production costs for a monopoly).

The analysis in Section 3 establishes these facts in greater generality, laying out the assumptions leading to efficiency gains in the segmented market, and providing intuition for the results.

3 The Main Results

Output can be thought of as some measure of gas, water, or telephone service. The cost function is given by $C(q) = F + c(q)$, where q denotes output, $F > 0$ is the fixed cost of creating and maintaining the distribution network, and $c(q)$ is the variable cost function. Let $p(q)$ be a decreasing market inverse demand function.

The market can be served by a single firm, and in this case there is an efficiency loss, the well-known deadweight loss of monopoly. Alternatively, a firm can bear the cost of creating and maintaining the distribution network, and then charge other firms for the network's usage. Roughly, under convexity of the variable cost function, $c(q)$, this market structure will be more efficient than the monopoly. As illustrated by the example in Section 2, the convexity of $c(q)$ does not preclude cost functions that have been used to define "natural monopolies," namely, those for which the minimizer of the average cost function, $AC(q) \equiv \frac{C(q)}{q}$, lies to the right of the maximum market demand, i. e. the q for which $p(q) = 0$, or, alternatively, strictly subadditive cost functions (recall eq. (1)).

Below we also consider the case when two or more firms share the cost of creating and maintaining the distribution network in the "upstream" market.

A monopoly is viable if there is a solution $q_{mon} > 0$ to the problem

$$\max_{q \ge 0} p(q)q - C(q) \tag{2}$$

that yields positive profits.

Now consider a firm, call it firm M, that owns the distribution network and charges other firms a price for each unit of "throughput," i. e. some measure of successful service delivery over the distribution network. Let p_M be the unit price charged by firm M, and suppose that the "downstream" market is populated by n firms, which compete in quantities *à la* Cournot. An output profile $(q_1, ..., q_n) \in \mathbb{R}^n_+$ is a *Cournot-Nash equilibrium* in the downstream market if

$$p(q_1 + ... + q_n)q_i - c(q_i) - p_M q_i \ge p\left(y_i + \sum_{j \ne i} q_j\right) y_i - c(y_i) - p_M y_i$$

for each $y_i \geq 0$ and $i \in \{1,\dots,n\}$. Suppose that a symmetric Cournot-Nash equilibrium profile $(q_1^*,\dots,q_n^*)$ with $q_1^* = \cdots = q_n^* =: q^*(n,p_M)$ exists.[4] Suppose further that the equilibrium entails positive output in the downstream market, i. e. $q^*(n,p_M) > 0$.[5] Then, if the profit function is differentiable, $q^*(n,p_M)$ satisfies

$$p'(nq^*(n,p_M))q^*(n,p_M) + p(nq^*(n,p_M)) - c'(q^*(n,p_M)) - p_M = 0. \tag{3}$$

Define $p^*(n,p_M) := p(nq^*(n,p_M))$ as the Cournot-Nash equilibrium price. If market demand is bounded (i. e. there exists $q > 0$ such that $p(q) = 0$) then $\lim_{n\to\infty} q^*(n,p_M) = 0$. Consequently, in view of eq. (3), if $p'(\cdot)$ has an upper bound, it follows that

$$\lim_{n\to\infty} p^*(n,p_M) = p_M + \lim_{n\to\infty} c'(q^*(n,p_M)) = p_M + \lim_{q\to 0} c'(q) = p_M + c_0,$$

where

$$c_0 \equiv \lim_{q\to 0} c'(q).$$

Thus, if there is enough competition in the downstream market, the equilibrium price,

$$p(nq^*(n,p_M)) = p_M + c'(q^*(n,p_M)) - p'(nq^*(n,p_M))q^*(n,p_M), \tag{4}$$

is close to $p_M + c_0$. Given p_M, demand in the upstream market is given by $q_M = nq^*(n,p_M)$. Hence, in view of eq. (4), the inverse demand function faced by firm M in the upstream market, $p_M(q_M)$, is given by

$$p_M(q_M) = p(q_M) - c'\left(\frac{q_M}{n}\right) + p'(q_M)\frac{q_M}{n}. \tag{5}$$

For large enough n, this function approximates the graph of $p(q) - c_0$ uniformly (over the range of nonnegative prices). If the inverse demand function $p_M(q_M)$ is differentiable, the first-order condition for firm M at an interior optimum point $q_M^*(n)$ is

$$p_M'(q_M^*(n))q_M^*(n) + p_M(q_M^*(n)) = 0.$$

The first main result (Proposition 1 below) rests on the following observations. First, if the marginal cost function $c'(q)$ is nondecreasing, then, for any large enough n, the upstream market is viable, in the sense that firm M's profits at $q_M^*(n)$ are positive. Second, if the marginal cost function $c'(q)$ is nondecreasing, firm profits in the downstream market are positive. Third, if the marginal cost function $c'(q)$ is nondecreasing and $c'(q_{mon}) > c_0$ (recall that q_{mon} is the monopoly output, i. e. a solution to eq. (2)), then, for any large enough n, the "segmented" market structure generates a lower deadweight loss than a monopoly.

To see that both markets are viable, suppose that the marginal cost function $c'(q)$ is nondecreasing. We have seen that if the number of firms in the downstream market, n, is sufficiently large, the market inverse demand curve faced by firm M can be taken arbitrarily close (with respect to the sup norm) to $p(q) - c_0$. Consequently, the firm M's profits at output level q_M are approximately $(p(q_M) - c_0)q_M - F$. Hence, letting $\pi_M^n(q_M)$ denote firm M's profits at output level q_M (given n firms in the downstream market), for a profit maximizer $q_M^*(n)$, and for any large enough n,

$$\pi_M^n(q_M^*(n)) \geq \pi_M^n(q_{mon}) \approx (p(q_{mon}) - c_0)q_{mon} - F \geq p(q_{mon})q_{mon} - c(q_{mon}) - F > 0,$$

where the second inequality follows from the fact that $c'(q)$ is nondecreasing, and the last inequality holds because, by assumption, q_{mon} yields positive monopoly profits.

Profits for a firm in the downstream market are given by

$$\begin{aligned}[p_M^*(n) + c'(q^*(n,p_M^*(n))) \quad & -p'(nq^*(n,p_M^*(n)))q^*(n,p_M^*(n))]q^*(n,p_M^*(n)) \\ & -c(q^*(n,p_M^*(n))) - p_M^*(n)q^*(n,p_M^*(n)),\end{aligned}$$

where $p_M^*(n) \equiv p_M(q_M^*(n))$. Rearranging terms gives

$$\begin{aligned}[c'(q^*(n,p_M^*(n))) \quad & -p'(nq^*(n,p_M^*(n)))q^*(n,p_M^*(n))]q^*(n,p_M^*(n)) - c(q^*(n,p_M^*(n))) \\ & = [c'(q^*(n,p_M^*(n))) - p'(nq^*(n,p_M^*(n)))q^*(n,p_M^*(n))]q^*(n,p_M^*(n)) \\ & - \int_0^{q^*(n,p_M^*(n))} c'(q)dq.\end{aligned}$$

Since $c'(q)$ is nondecreasing and $p'(\cdot) < 0$, it follows that this expression is positive.

Finally, as n tends to infinity, $q_M^*(n)$ converges to a point $q^\bullet$ such that $p(q^\bullet) + p'(q^\bullet)q^\bullet = c_0$ (recall that the inverse demand function faced by firm M uniformly approximates the function $p(q) - c_0$). Since, by assumption, $c'(q_{mon}) > c_0$, it follows that $q^\bullet > q_{mon}$. Consequently, for large enough n, $q_M^*(n)$ generates a lower deadweight loss than the monopoly quantity q_{mon}.[6]

The assumptions and the main conclusions are collected in the following proposition.

Proposition 1

Assume the following:

- *The market inverse demand function p(q) is decreasing, twice differentiable, and bounded in the sense that there exists $q > 0$ such that $p(q) = 0$.*
- *The map $q \mapsto p(q) + p'(q)q$ is bounded.*
- *The variable cost function c(q) is increasing, twice differentiable, and convex, with $c'(q_{mon}) > c_0 \equiv \lim_{q\to 0} c'(q)$ (where q_{mon} is a solution to eq. (2)).*

If a monopoly is viable (i. e. if there is a solution to eq. (2) that yields positive profits), then a market structure with a monopolist in the upstream market and a sufficiently large number of firms in the downstream market is viable (in the sense that all firms make positive profits). Moreover, output in the downstream market converges, as the number of firms grows large, to a point $q^\bullet$ such that $p(q^\bullet) + p'(q^\bullet)q^\bullet = c_0$. Since $c'(q_{mon}) > c_0$ implies $q^\bullet > q_{mon}$, this market structure yields a lower deadweight loss than a monopoly.

Remark

The reader may wonder whether it would be in the interest of a monopoly to "unbundle" production by means of the segmented market structure considered in Proposition 1. The answer is in the affirmative, provided that $c'(q_{mon}) > c_0$ and there is enough competition in the downstream market. Indeed, as the number of downstream firms, n, increases, (i) firm markups in the downstream market vanish, thereby eliminating double marginalization; and (ii) downstream firms can produce at a lower cost than a monopoly.

Discussion

The overall efficiency of the market structure considered here may not be higher than that for the standard average cost monopoly pricing rule: depending on the shape of the demand and cost functions, examples can be constructed in which the segmented market considered in Proposition 1 (resp. the average cost pricing rule) outperforms the average cost pricing rule (resp. the segmented market considered in Proposition 1). However, the informational requirements of the average cost pricing rule are significant: the regulator must know both the cost function and the demand function. By contrast, the segmented market structure considered here requires only that the firms know their cost function and the market demand function. In addition, for n large, firm M does not even need to know the function $c(q)$. Indeed, in this case M can infer c_0 from the equilibrium prices in both markets, since for large n the equilibrium price in the downstream market is approximately $p_M + c_0$. Furthermore, policies favoring market segmentation need not preclude price regulation. In fact, once the market is segmented, regulation of the price charged by the upstream monopolist may be considered.

The literature on access pricing (*cf.* Armstrong 2002; Laffont and Tirole 2000) considers an upstream market operated by a monopolist, which supplies access services to the downstream (retail) firms. This literature focuses entirely on regulating access charges in the case when the monopolist also operates in the retail market (the case of vertical integration). Armstrong, Cowan, and Vickers (1994, Section 5.2.1) and Laffont and Tirole (2000, Section 2.2.5) briefly touch upon the case of vertical separation, which is related to the market structure considered here, and which reduces, in the present context, to the case when access charges are set so that the monopolist breaks even. This pricing rule makes strong assumptions on the information available to the regulator.

Other authors have studied ownership structure in industries where naturally monopolistic and competitive activities are vertically related. For example, Cremer and De Donder (2013) take the case of vertical separation (i. e. the situation in Proposition 1, which they call ownership unbundling), with competition in the downstream market, as the point of departure, and ask whether so-called legal unbundling—an ownership structure whereby the upstream firm chooses the size of the network to maximize the sum of its profits and those of a downstream subsidiary firm—can lead to a social welfare improvement.

To conclude this discussion, we comment on the relationship between the market structure considered in Proposition 1 and the notion of vertical foreclosure studied in Rey and Tirole (2007). Vertical foreclosure refers to situations in which an upstream monopolist, which produces a key input for downstream use, deliberately restricts competition in the downstream segment of the market, favoring just a few downstream firms, by denying proper access to the input to potential downstream competitors.

Since the presence of vertical foreclosure may hamper the welfare-improving market structure considered in this paper—which relies, in a crucial way, on the existence of competition in the downstream segment of the market—it is important to understand the conditions under which this phenomenon is more likely to occur.

First, we show that, in our setting, vertical foreclosure will not arise. The absence of vertical foreclosure is consistent with the Chicago School critique (see Rey and Tirole 2007 and references therein), which questions the rationale for excluding downstream competitors. Recall that the inverse demand function faced by the upstream monopolist, M, is given by eq. (5). The partial derivative of the right-hand-side of eq. (5) with respect to n is

$$\frac{q_M}{n^2}c''\left(\frac{q_M}{n}\right) - p'(q_M)\frac{q_M}{n^2}.$$

Since $c(\cdot)$ is convex and $p(\cdot)$ is decreasing, this expression is positive. Consequently, an increase in the number of active firms in the downstream market shifts the upstream monopolist's inverse demand curve, $p_M(q_M)$, outward. Therefore, since M's profits at q_M are given by $p_M(q_M)q_M - F$, it follows that the upstream monopolist's profits are positively correlated with the degree of competition in the downstream market, as measured by n. Thus, the upstream monopolist benefits from downstream competition and, to the extent that downstream competition improves social welfare, the monopolist's objectives in the segmented market do not conflict with those of a benevolent planner.[7]

Recall that an increase in the number of active firms in the downstream segment of the market, n, shifts the upstream monopolist's inverse demand curve, $p_M(q_M)$, outward. Whether or not this outward shift translates in a larger industry output, and hence a lower deadweight loss, depends on whether the upstream monopolist expands output in response to increased competition in the downstream segment of the market.

The first-order condition for M at an interior optimum point $q_M > 0$ sets the marginal revenue curve equal to zero:

$$p'_M(q_M)q_M + p_M(q_M) = 0.$$

Using eq. (5), the marginal revenue curve is expressible as follows:

$$p'(q_M)q_M - c''\left(\frac{q_M}{n}\right)\frac{q_M}{n} + p''(q_M)\frac{q_M^2}{n} + \frac{q_M}{n}p'(q_M) + p(q_M) - c'\left(\frac{q_M}{n}\right) + p'(q_M)\frac{q_M}{n}.$$

Differentiating with respect to n gives

$$\frac{q_M}{n^3}c'''\left(\frac{q_M}{n}\right) + 2\frac{q_M}{n^2}c''\left(\frac{q_M}{n}\right) - \frac{q_M}{n^2}\left(p'(q_M) + p''(q_M)q_M\right) - p'(q_M)\frac{q_M}{n^2}. \tag{6}$$

Because $c(\cdot)$ is convex and $p'(\cdot) \leq 0$, this expression is positive if

$$p'(q_M) + p''(q_M)q_M \leq 0, \quad \text{for each } q_M, \tag{7}$$

and $c'''(\cdot) \geq 0$ (i. e. the marginal cost curve is convex). In this case, an increase in n results in an outward shift of the monopolists marginal revenue curve, implying a larger equilibrium industry output, and, in turn, a lower deadweight loss. The condition eq. (7) is standard in the literature (see, e. g. Novshek (1985, Theorem 3, Condition (3)), Vives (1999, § 4.2), and Yi (1998, Assumption 2)) and ensures that the marginal revenue curve is decreasing. While assuming convexity of the marginal cost curve seems restrictive, it is worth pointing out that this assumption can be dropped for large n. Indeed, note from the derivative in eq. (6) that, even when the third derivative $c'''(\cdot)$ is negative, the effect of the first term in eq. (6) is an order of magnitude smaller, for large n, than that of the other terms in eq. (6).

Having considered the case of a monopoly in the upstream market, we turn to a market structure in which two or more firms share the fixed cost F and compete for service in the upstream market. It will be shown that this arrangement improves the overall market efficiency.

For $m \geq 2$, assume that an m-firm oligopoly with constant marginal cost c_0 and fixed cost sharing is viable, i. e. consider the symmetric m-firm Cournot game in which firm i's profit at output profile $(q_1, ..., q_m)$ is given by

$$p(q_1 + ... + q_m)q_i - c_0 q_i - \frac{F}{m},$$

and suppose that any symmetric output profile $(q, ..., q) \gg 0$ satisfying the first-order condition for an interior symmetric Cournot-Nash equilibrium,

$$p(mq) + p'(mq)q = c_0,$$

yields positive profits for each firm.[8] Let P be the price charged by the firms in the upstream market. As before, the n firms in the downstream market compete in quantities *à la* Cournot. An interior output per firm at a symmetric Cournot-Nash equilibrium in the downstream market, $q^*(n, P) > 0$, satisfies the analogue to eq. (3):

$$p'(nq^*(n,P))q^*(n,P) + p(nq^*(n,P)) - c'(q^*(n,P)) - P = 0.$$

From the previous analysis, the inverse demand function faced by the firms in the upstream market is given by

$$P(Q) = p(Q) - c'\left(\frac{Q}{n}\right) + p'(Q)\frac{Q}{n}.$$

An output profile $(Q_1, ..., Q_m) \in \mathbb{R}_+^m$ is a Cournot-Nash equilibrium in the upstream market if

$$P(Q_1 + ... + Q_m)Q_i \geq p\left(y_i + \sum_{j \neq i} Q_j\right)y_i$$

for each $y_i \geq 0$ and $i \in \{1,...,m\}$. Let $(Q^*(m,n), ..., Q^*(m,n))$ be a symmetric Cournot-Nash equilibrium profile.[9] Suppose that $Q^*(m,n) > 0$.[10] Then $Q^*(m,n)$ satisfies

$$P'(mQ^*(m,n))Q^*(m,n) + P(mQ^*(m,n)) = 0.$$

For large n, $P(Q)$ uniformly approximates (given $p'(\cdot)$ bounded) $p(q) - c_0$, and so it follows that for large n, $Q^*(m,n)$ approximates $Q^*(m)$ such that

$$p(mQ^*(m)) + p'(mQ^*(m))Q^*(m) = c_0. \tag{8}$$

For $m \geq 2$, one has $\overline{Q}(m) \equiv mQ^*(m) > q^\bullet$, where $q^\bullet$ is defined by $p(q^\bullet) + p'(q^\bullet)q^\bullet = c_0$. Indeed, the map $Q \mapsto p(Q) + p'(Q)\frac{Q}{m}$ is increasing in m, for

$$\frac{\partial\left(p(Q) + p'(Q)\frac{Q}{m}\right)}{\partial m} = -p'(Q) \cdot \frac{Q}{m^2} > 0,$$

and so $\overline{Q}(m) > q^\bullet$ for $m \geq 2$. Therefore, since $q^\bullet > q_{mon}$ by Proposition 1, it follows that for any large enough n, $mQ^*(m,n)$ yields a lower deadweight loss than the monopoly quantity q_{mon}. Moreover, $\overline{Q}(m+1) > \overline{Q}(m)$ for any $m \geq 2$, so that the deadweight loss decreases with m.

If market demand is bounded (i. e. there exists $q > 0$ such that $p(q) = 0$) then $\lim_{m\to\infty} Q^*(m) = 0$ and, in view of eq. (8), $\lim_{m\to\infty} p(\overline{Q}(m)) = c_0$. Thus, as m grows large, the deadweight loss vanishes.

On the other hand, because $Q^*(m)$ satisfies eq. (8), the viability assumption implies that

$$p(mQ^*(m))Q^*(m) - c_0Q^*(m) - \frac{F}{m} > 0.$$

But profits per firm in the upstream market converge, as n grows large, to the left-hand side of this inequality, and so it follows that upstream firms make positive profits for large n. The viability of downstream firms can be established as in the proof of Proposition 1.

For the case of cost sharing in the upstream market, the following result has been obtained.

Proposition 2. *Assume the following:*

- *The market inverse demand function $p(q)$ is decreasing, twice differentiable, and bounded in the sense that there exists $q > 0$ such that $p(q) = 0$.*
- *The map $q \mapsto p(q) + p'(q)q$ is bounded.*
- *The variable cost function $c(q)$ is increasing, twice differentiable, and convex, with $c'(q_{mon}) > c_0 \equiv \lim_{q\to 0} c'(q)$ (where q_{mon} is a solution to eq. (2)).*

If an m-firm oligopoly with constant marginal cost c_0 and fixed cost sharing is viable, then a market structure with m firms in the upstream market and a sufficiently large number of firms in the downstream market is viable (in the sense that all firms make positive profits) and yields a lower deadweight loss than a monopoly. Moreover, the deadweight loss is decreasing with m, and vanishes for large m.

Acknowledgements

This paper benefited from valuable comments provided by Tomas Sjöström and an anonymous referee.

Notes

1 See, e. g. Braeutigam (1989).
2 In the literature, the definition of a natural monopoly tends to agree with the notions presented above. However, some authors work with slightly different concepts. For example, Kreps (2004) is careful enough to require not only decreasing average (or strictly subadditive) costs, but also marginal costs that are "constant or falling for levels that are large relative to market demand," in addition to substantial fixed costs. Our results require increasing marginal costs, and so they do not cover Kreps' definition.
3 The cost efficiency argument fails to account for the deadweight loss of monopoly, which exceeds that for an industry in which two or more firms compete to sell their output. From a welfare perspective, competition may be desirable in spite of the cost efficiency loss associated with strictly subadditive cost functions.
4 A symmetric Cournot-Nash equilibrium exists under the following assumptions (*cf*. McManus 1964; Novshek 1985, Theorem 1): (i) $p : [0, +\infty) \to \mathbb{R}_+$ is nonincreasing and upper semicontinuous, and the total revenue function is bounded; and (ii) $c : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous, increasing, and convex. 4.
5 It will be seen later that under this further assumption, and for any large enough n, the firm M sets its price approximately equal to $p(q^\bullet) - c_0$, where $c_0 \equiv \lim_{q\to 0} c'(q)$ and $q^\bullet > q_{mon} > 0$ satisfies $p(q^\bullet) + p'(q^\bullet)q^\bullet = c_0$. At this price, there is no equilibrium in the downstream market in which all firms shut down. Indeed, because $p'(\cdot) < 0$, so that $p(0) > p(q^\bullet)$, a downstream firm best responds to a zero output profile by choosing a positive output level.
6 As is standard in the literature, here we approximate the deadweight loss associated with a given market allocation by the sum of consumer and producer surplus forgone, relative to an efficient resource allocation. As is well-known, this measure is precise enough if income effects are sufficiently small. Vives (1987) formalizes Marshall's (1920) idea that income effects are small when the proportion of income spent on any commodity is small.
7 In a framework with incomplete information and nonlinear pricing, Rey and Tirole (2007) show that vertical foreclosure may occur if the upstream monopolist cannot commit to a pricing policy.
8 Note that if $p'(\cdot) < 0$ and $c'(q)$ is nondecreasing, then firms make profits only if $p(0) > c_0$. 8.
9 If $p(q)$ is continuous and decreasing, and there exists q such that $p(q) = 0$, then the existence of such an equilibrium follows from the Theorem in Roberts and Sonnenschein (1976).
10 This assumption is, in fact, without loss of generality, at least for large enough n. Indeed, for large n, $P(Q)$ uniformly approximates (given $p'(\cdot)$ bounded) $p(q) - c_0$, and since $p(0) > c_0$ (see footnote 8), it follows that an upstream firm's best response to a zero output profile is a positive output level.

References

Armstrong, M., S. Cowan, and J. Vickers. 1994. *Regulatory Reform: Economic Analysis and British Experience*. Cambridge, MA: The MIT Press.
Armstrong, M. 2002. "The Theory of Access Pricing and Interconnection." In *Handbook of Telecommunications Economics, Volume 1*, edited by M. E. Cave, S. K. Majumdar, and I. Vogelsang. Amsterdam: Elsevier.
Braeutigam, R. R. 1989. "Optimal Policies for Natural Monopolies." In *Handbook of Industrial Organization, Volume II*, edited by R. Schmalensee, and R. D. Willig. Amsterdam: Elsevier Science B.V.
Cremer, H., and P. De Donder. 2013. "Network Investment under Legal and Ownership Unbundling." *Review of Network Economics* 12: 27–59.
Kreps, D. M. 2004. *Microeconomics for Managers*. New York: W.W. Norton & Company.
Laffont, J.-J., and J. Tirole. 2000. *Competition in Telecommunications*. Cambridge, MA: The MIT Press.
Marshall, A. 1920. *Principles of Economics*. London: Macmillan.
McManus, M. 1964. "Equilibrium, Numbers and Size in Cournot Oligopoly." *Yorkshire Bulletin of Social and Economic Research* 16: 68–75.
Novshek, W. 1985. "On the Existence of Cournot Equilibrium." *Review of Economic Studies* 52: 85–98.
Rey, P., and J. Tirole. 2007. "A Primer on Foreclosure." In *Handbook of Industrial Organization, Volume III*, edited by M. Armstrong, and R. Porter. Amsterdam: Elsevier B.V.
Roberts, J., and H. Sonnenschein. 1976. "On the Existence of Cournot Equilibrium without Concave Profit Functions." *Journal of Economic Theory* 13: 112–17.
Vives, X. 1987. "Small Income Effects: A Marshallian Theory of Consumer Surplus and Downward Sloping Demand." *Review of Economic Studies* 54: 87–103.
Vives, X. 1999. *Oligopoly Pricing: Old Ideas and New Tools*. Cambridge, MA: The MIT Press.
Yi, S-S. 1998. "Endogenous Formation of Joint Ventures with Efficiency Gains." *Rand Journal of Economics* 29: 610–31.