

Poverty-Reducing Taxation with Endogenous Income^{*}

Oriol Carbonell-Nicolau[†]

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Abstract

This paper develops nonlinear income tax systems designed to reduce poverty within a framework where income is endogenously determined. Our approach to poverty reduction is applicable across a wide range of poverty measures and remains robust under varying economic conditions, including heterogeneous wage distributions, non-labor endowments, and diverse individual preferences. When negative marginal tax rates are deemed infeasible—potentially due to concerns about vertical equity—a poverty-reducing schedule must provide a uniform lump-sum subsidy across the range where net income can remain weakly below the poverty line. Even when negative marginal tax rates are permissible, effective poverty-reducing tax policies must still rely on subsidization for the poor and non-positive marginal tax rates as essential features.

Keywords: poverty measurement, nonlinear income taxation, poverty-reducing taxation.

JEL classifications: D63, D71, I32.

1. Introduction

Unlike the traditional utilitarian approach prevalent in neoclassical economics, this paper employs a non-welfarist framework for policy evaluation. We explore the design of poverty-focused, nonlinear income tax systems, evaluating their effectiveness across diverse economic contexts.

The framework we develop emphasizes three core considerations of the underlying economic problem. First, we treat income as endogenous, recognizing that individuals adjust their earnings in response to tax policies, available non-labor income, and labor market opportunities. Second, we demonstrate that the impact of tax policy on poverty remains qualitatively consistent across varying economic conditions and heterogeneous individual preferences—elements that are either dynamic or unobservable. Third, our theoretical framework applies to a broad spectrum of established poverty measures, enhancing its practical relevance.

While the traditional welfarist approach requires the government to aggregate individual utility functions—unobservable theoretical constructs—into a social welfare function that

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[†]Department of Economics, Rutgers University, 75 Hamilton St., New Brunswick, NJ 08901. E-mail: carbonell-nicolau@rutgers.edu.

makes explicit judgments about interpersonal utility comparisons (see [Boadway, 2012](#)), our poverty-focused framework provides a more concrete and measurable policy objective. This shift toward an observable and widely understood policy target offers theoretical clarity and practical advantages for policy design and evaluation.

Our analysis builds on the classical Mirrlees model of endogenous income ([Mirrlees, 1971](#)), extending it to incorporate two additional dimensions of heterogeneity beyond the standard variation in individual wage rates.

First, the framework allows for heterogeneity in individual preferences. Admissible preferences are represented by idiosyncratic utility functions defined over labor-consumption bundles, which are assumed to satisfy standard regularity conditions. A central assumption is the normality of consumption, which implies that consumption increases in response to lump-sum subsidies, holding other factors constant.

Second, the model introduces heterogeneity in non-labor income through individual-specific endowments. This variation captures differences in unearned income across individuals, further enriching the economic environment.

Poverty measurement has generated extensive scholarly literature encompassing diverse methodologies for complete and incomplete poverty orderings of income distributions. Despite this methodological variety, economists broadly agree that poverty measures must conform to certain fundamental principles.¹

Our theoretical framework builds upon four essential axioms from this literature. *Focus* requires poverty measurement to remain unchanged when only the incomes of the non-poor change. *Symmetry* ensures that poverty measurement remains independent of individual characteristics beyond income. *Continuity* requires that marginal changes in income levels produce correspondingly small changes in poverty measurements. Finally, *Monotonicity* establishes that ceteris paribus, any decrease in a poor person's income must result in an increased poverty measure. These principles belong to the axiomatic tradition initiated by [Sen \(1976\)](#) and provide the baseline requirements used in our analysis. Our main characterization applies to the unanimity preorder generated by these principles and, as shown below, to every admissible poverty ordering extending it.

We consider a multidimensional tax policy space where an *income tax schedule* is a piecewise linear mapping from gross incomes to tax liabilities. These policies depend solely on observed gross income and cannot condition on unobserved individual characteristics such as wage rates, non-labor endowments, or preferences. Additionally, a valid income tax schedule must satisfy three fundamental properties—continuity, rank preservation, and monotonicity—while ensuring that tax liabilities never exceed gross incomes. Continuity prevents cliff effects that distort behavior and violate horizontal equity. Rank preservation ensures the relative income ordering of individuals remains unchanged post-taxation. Monotonicity embodies vertical equity, stipulating that higher-income individuals face weakly higher tax liabilities. While this monotonicity precludes negative marginal tax rates—a constraint extensively studied in optimal taxation—our extended analysis later relaxes this assumption, revealing that negative marginal tax rates can emerge as effective poverty-reducing tools.

A *poverty-reducing tax schedule* ensures that, across all economic environments—regardless of non-labor endowment or wage distributions and individual preference profiles—the after-tax income distribution exhibits no more poverty than the tax-free distribution.

¹For a comprehensive review of these principles, see [Zheng \(2000\)](#).

This robustness makes poverty reduction a consistent policy goal despite dynamic or unobservable economic factors.

We provide a comprehensive characterization of poverty-reducing tax schedules. For a given poverty line $z > 0$, let $t(y)$ denote the tax liability at gross income y and let $s = -t(0)$ denote the subsidy at zero income. The schedule is poverty-reducing if and only if $s \geq 0$ and it provides the same subsidy s at every gross-income level y satisfying $y + s \leq z$.

This characterization reflects a simple insight about the interaction between taxation and poverty. If a schedule fails to subsidize the poor or imposes a positive marginal tax rate over the poverty-relevant income range, one can construct a homogeneous economy in which every individual attains higher consumption without taxation than under the tax schedule. Taxation therefore increases poverty in that economy.

Beyond offering a clear and implementable criterion for tax policy design, this characterization yields several insights.

First, the poverty-reduction property defined in this paper is context-free: it cannot be contingent on resource constraints, which fluctuate with the underlying model fundamentals. Our analysis identifies the essential characteristics of poverty-reducing tax policies, providing a robust theoretical foundation upon which more specific, context-dependent policies can be built.

Second, the universality of our results extends beyond context-independence with respect to endowment and wage distributions to encompass essentially the entire class of admissible preference profiles. This feature stems from the pivotal role of homogeneous societies in our proofs: any tax schedule violating the uniform-subsidy characterization can be shown to increase poverty in a homogeneous environment using particular standard preferences. Our complete characterization requires only that the admissible preference domain contain the simple quasilinear functional forms used in the necessity argument; beyond this, it places no restrictions on preference heterogeneity across individuals.

This preference robustness is absent from inequality- and polarization-reducing tax design, which requires not only the classical single-crossing condition (Mirrlees, 1971) but additional order-preserving restrictions on the wage elasticity of income (Carbonell-Nicolau and Llavador, 2018, 2021a,b, 2025; Carbonell-Nicolau, 2019, 2024, 2025). Our framework is free of such constraints.

Third, the set of poverty-reducing tax schedules admits a complete preorder based on the poverty-relevant subsidy $\sigma_z(t) = \min\{-t(0), z\}$. Larger poverty-relevant subsidies yield weakly lower poverty under every admissible measure satisfying Focus, Symmetry, Continuity, and Monotonicity; once the subsidy reaches z , further increases are immaterial because the censored distribution has attained its maximal value. This complete preorder enables straightforward comparisons between poverty-reducing tax schedules beyond their poverty dominance relative to the tax-free benchmark.

Fourth, when the monotonicity requirement for tax schedules is relaxed, a broader class of poverty-reducing policies emerges: tax liabilities must be non-positive on $[0, z]$ and non-increasing over the poverty-relevant set of gross incomes that yield net incomes strictly below the poverty line.

Removing the monotonicity constraint opens the door to subsidizing the poor through negative marginal tax rates. Under such a system, subsidies can increase as individuals earn more income, creating an incentive structure that rewards additional work. This approach aligns with findings in the broader literature, which we explore in greater detail below.

However, despite this expanded policy space, one fundamental principle from our earlier analysis remains unchanged: attempting to subsidize the poor while simultaneously imposing positive marginal tax rates over the poverty-relevant income range is ineffective at reducing poverty.

Finally, we maintain rank preservation, requiring that higher gross income never yield lower net income, as a modeling assumption for normative and analytical reasons. This restriction is stronger than behavioral relevance alone requires, since rank-violating portions of a net-income schedule are behaviorally irrelevant whenever they are dominated by attainable bundles at lower gross-income levels.

Beyond these core insights, we test the robustness of our characterization in three directions. First, we examine alternative poverty measurement frameworks to assess how our results depend on the adopted axiomatic structure. Second, we investigate institutional and informational constraints—such as endogenous poverty lines or multiple contested thresholds—that complicate real-world implementation. Third, we connect our outcome-based approach to normative principles from taxation theory, revealing both convergences and tensions.

Regarding poverty measurement, we analyze distribution-sensitive criteria that incorporate inequality concerns among the poor—building on [Shorrocks's \(1983\)](#) relative and absolute generalized Lorenz criteria, applied to censored income distributions. Here, the requirement of uniform lump-sum subsidies remains necessary: any non-fixed subsidy can increase poverty even under these refined measures, as demonstrated by homogeneous economies where inequality among the poor is absent. However, the sufficiency argument underlying our core criterion does not automatically carry over. Heterogeneous responses to a uniform subsidy can worsen inequality among the poor even while raising individual incomes, creating an obstruction to poverty dominance under distribution-sensitive criteria.

On informational and institutional constraints, we first address the case of multiple poverty lines. We show that robustness across multiple poverty lines is governed by the highest threshold under consideration. Letting $s = -t(0)$, the schedule must provide the uniform subsidy s whenever $y + s \leq \sup Z$; if $s \geq \sup Z$, rank preservation places every attainable net income weakly above every threshold in Z . Building on this result, we can extend the analysis to endogenously determined poverty lines—such as thresholds defined as a fixed fraction of median income—with one important caveat. If poverty lines depend on post-tax incomes, a fundamental circularity arises whereby poverty status depends on the very tax policy being evaluated. However, this circularity dissolves when the threshold anchors to predetermined reference statistics (e.g., lagged medians). In such cases, letting $s = -t(0)$, the subsidy must remain uniform over gross incomes up to $\sup Z - s$ whenever this range is nonempty; if $s \geq \sup Z$, every attainable net income is weakly above every reference threshold. This prescription preserves our core robustness desideratum—even when the poverty line calibrates to historical or projected fundamentals, the actual economic environment at implementation (wage distributions, endowments, preferences) remains uncertain.

The critical role of endogenous income emerges most clearly when comparing our framework with its exogenous counterpart. With exogenous income, any subsidy to the poor that preserves non-poor status guarantees universal poverty reduction. When income is endogenous, however, incentive-compatibility constraints restrict the viable policies to uniform subsidies if negative marginal tax rates are excluded. Non-uniform subsidies may induce labor-supply responses that reduce post-tax income below its tax-free level.

Finally, we bridge our framework to two alternative normative principles.

Adapting Young's (1987; 1988; 1990) classical *equal sacrifice* principle from taxation theory, we identify conditions under which poverty-reducing subsidies deliver *equal benefit* (identical utility gains) to all individuals remaining poor post-subsidy. Under quasilinear preferences, uniform lump-sum subsidies implement equal benefit by eliminating income effects on labor supply. We also use graphical arguments to explain how income effects and bunching at the boundary of the uniform-subsidy region can make utility gains vary across types, highlighting the contrast between outcome-based poverty reduction and utility-based fairness.

We also introduce *poverty preservation*—the requirement that taxation not reverse preexisting poverty orderings across societies—as a cross-society analogue of rank preservation. Among strictly rank-preserving schedules, poverty preservation implies poverty reduction. The converse need not be expected to hold: although uniform lump-sum subsidies satisfy universal poverty reduction, heterogeneous income effects create a channel through which poverty rankings across societies may be reversed.

1.1. Literature on poverty and taxation

The literature on income taxation and poverty has diverged into two primary research trajectories. The first integrates poverty concerns into the classical utilitarian framework of optimal taxation, part of a broader scholarly effort to incorporate fairness principles into traditional social welfare theory (Fleurbaey and Maniquet, 2018). The second, which we follow, departs from traditional welfarism by placing poverty alleviation at the center of the analysis as an explicit policy objective.

Within the first tradition, poverty concerns have been incorporated into optimal taxation through generalized social marginal welfare weights (Saez and Stantcheva, 2016). These weights can assign greater normative importance to gains accruing to individuals below a poverty threshold while retaining the local-reform methodology of optimal tax theory. However, Sher (2024) shows that when such weights depend on policy outcomes, they generally need not represent the gradient of a global social welfare function. The resulting local comparisons may therefore fail to generate a coherent global social ordering unless the weights satisfy restrictive integrability conditions.

Our framework takes a different route. Because the poverty criterion is specified directly as a global preorder over post-tax income distributions, the analysis does not rely on integrating policy-dependent marginal welfare weights. Other welfarist approaches formulate well-defined social orderings that combine poverty reduction with efficiency and responsibility-sensitive fairness (Henry de Frahan and Maniquet, 2021; Maniquet and Neumann, 2021). A more detailed comparison with responsibility-sensitive welfarism, including points of convergence concerning low-income marginal tax rates, appears in Section 4.2.9.

We now review the literature that, like our paper, takes poverty alleviation as its primary policy objective.

Besley and Coate (1992) and Besley and Coate (1995) provide a positive foundation for income-based poverty analysis, arguing that poverty programs funded by general taxation reflect taxpayers' concern with visible income deprivation rather than recipients' unobserved leisure choices. This justifies evaluating anti-poverty policy directly on the income distribution—a premise our paper shares.

Besley and Coate further characterize cost-minimizing income maintenance programs designed to bring all individuals above a poverty threshold, proposing work requirements for low-wage workers as a means of reducing public expenditure.

[Bourguignon and Fields \(1990\)](#) examine the features of a poverty-minimizing anti-poverty budget using complete orderings of income distributions. They find that the optimal allocation is sensitive to the specific poverty index adopted and the initial income distribution. By contrast, our characterization does not optimize subject to a fixed resource constraint; instead, it asks which tax schedules reduce poverty universally, across all possible environments and poverty measures satisfying our four core poverty axioms. The answer—uniform lump-sum subsidies—is therefore invariant to the choice of poverty measure and the initial distribution of fundamentals.

[Kanbur et al. \(1994\)](#) provide a partial characterization of tax schedules aimed at minimizing an income-based poverty index of the additively separable form considered by [Atkinson \(1987\)](#). They find that optimal marginal tax rates should be negative at the lower end of the income distribution—aligning with our results—though their characterization does not impose universality across fundamentals.

Building on this work, [Pirttilä and Tuomala \(2004\)](#) extend the analysis to encompass mixed tax systems, which involve the simultaneous taxation of income and commodities, while maintaining the explicit social objective of poverty minimization. Their findings demonstrate that this approach justifies differentiated tax rates across commodities, challenging the conventional wisdom established by [Atkinson and Stiglitz \(1976\)](#), which advocates for uniform commodity taxation.

[Chambers and Moreno-Ternero \(2017\)](#) develop an axiomatic approach to poverty-sensitive taxation by building on Young’s equal sacrifice principle ([Young, 1987, 1988, 1990](#)). They selectively remove certain axioms from Young’s original framework—specifically, those axioms that prevent tax systems from exempting low-income individuals—enabling a formal characterization of generalized equal-sacrifice tax rules that integrate poverty alleviation goals.

Finally, [Carbonell-Nicolau \(2026\)](#) also adopts a universal poverty reduction criterion, but focuses on distribution-sensitive poverty measures that need not satisfy the Monotonicity axiom. That paper completely characterizes poverty-reducing tax schedules within an exogenous income framework. The present paper shifts the focus to a non-nested class of poverty measures and extends the analysis to endogenous income, where behavioral responses fundamentally reshape the set of poverty-reducing policies.

The paper is structured as follows. [Section 2](#) defines the class of poverty measures for which our results are valid. [Section 3](#) introduces the endogenous income model and formulates the tax policy space. The main findings are presented in [Section 4](#), where we discuss the implications of our results and develop heuristic insights into the logic behind the characterization of poverty-reducing tax schedules. Finally, [Section 5](#) summarizes the main contributions and suggests directions for future research.

2. Poverty criterion

Our poverty criterion is censored first-order stochastic dominance, which captures unanimity across a broad class of conventional poverty measures.

An *income distribution* is formally defined as an n -dimensional vector

$$\mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}_+^n,$$

where each x_i denotes the income of individual i , and $n \in \mathbb{N}$ is the population size.

Let $(x_{[1]}, \dots, x_{[n]})$ be a non-decreasing rearrangement of the coordinates in $\mathbf{x}$:

$$x_{[1]} \leq \dots \leq x_{[n]}.$$

Given an income distribution $\mathbf{x} = (x_1, \dots, x_n)$ and a *poverty line* $z > 0$, we define the *poor* as the set of individuals i whose income x_i is less than or equal to z .

The distribution $\mathbf{x} = (x_1, \dots, x_n)$ *censored at* z , denoted as

$$\mathbf{x}^z = (x_1^z, \dots, x_n^z),$$

is defined by

$$x_i^z = \begin{cases} x_i & \text{if } x_i \leq z, \\ z & \text{if } x_i > z, \end{cases}$$

for each $i \in \{1, \dots, n\}$.

First-order stochastic dominance is a binary relation $\succsim_D$ over the set of income distributions. For any two distributions $\mathbf{x}$ and $\mathbf{y}$, we say that $\mathbf{x}$ dominates $\mathbf{y}$ (written as $\mathbf{x} \succsim_D \mathbf{y}$) if and only if

$$x_{[i]} \geq y_{[i]}, \quad \text{for each } i \in \{1, \dots, n\},$$

where $x_{[i]}$ and $y_{[i]}$ represent the i -th ordered elements of the respective distributions.

For a fixed poverty line $z > 0$, we establish a ranking system for income distributions using a poverty preorder $\succsim_z$. This binary relation over the set of distributions is defined by

$$\mathbf{x} \succsim_z \mathbf{y} \Leftrightarrow \mathbf{y}^z \succsim_D \mathbf{x}^z,$$

where $\mathbf{x}^z$ and $\mathbf{y}^z$ represent the censored distributions of $\mathbf{x}$ and $\mathbf{y}$, respectively, at the poverty line z .

The relation “ $\mathbf{x} \succsim_z \mathbf{y}$ ” should be interpreted as “the distribution $\mathbf{x}$ exhibits no less poverty than the distribution $\mathbf{y}$,” relative to the poverty line z .

We consider the class of poverty measures satisfying the following four principles:

1. *Focus*: Changes in non-poor incomes do not affect poverty measurement.
2. *Symmetry*: Poverty measurement remains unchanged when individuals exchange incomes.
3. *Continuity*: Poverty measurement varies continuously with income levels.
4. *Monotonicity*: A decrease in a poor person's income increases poverty.

For fixed population size, censored first-order stochastic dominance is the unanimity preorder associated with this class. More precisely, for any two income distributions $\mathbf{x}$ and $\mathbf{y}$,

$$\mathbf{x} \succsim_z \mathbf{y} \Leftrightarrow P(\mathbf{x}) \geq P(\mathbf{y}) \text{ for every admissible poverty measure } P,$$

where larger values of P represent greater poverty. Accordingly, $\succsim_z$ records exactly those comparisons on which all poverty measures satisfying Focus, Symmetry, Continuity, and Monotonicity agree. It may therefore leave incomparable distributions that a particular admissible poverty measure, such as a censored utilitarian poverty measure, is able to rank. This result was first established by Atkinson (1987) for the subclass of additively separable poverty measures. For a comprehensive discussion of these axioms and their implications, see Zheng (2000).

3. Endogenous income framework

Our model extends the foundational Mirrlees (1971) income taxation framework by incorporating both labor and non-labor income sources, as well as heterogeneous preferences.

Individual i 's preferences over labor-consumption pairs $(l, x) \in [0, 1] \times \mathbb{R}_+$ are represented by a utility function $u_i(l, x)$, which is non-increasing in labor, l , and non-decreasing in consumption, x , and strictly decreasing in l and strictly increasing in x over $(0, 1) \times \mathbb{R}_{++}$. The quantity of labor hours, l , is normalized to the range between zero and one, where $l = 0$ represents no work and $l = 1$ represents full-time work.

The utility function u is continuous over its domain $[0, 1] \times \mathbb{R}_+$ and differentiable and strictly quasiconcave on the interior of this domain. Furthermore, consumption is assumed to be normal: under a parallel outward shift of a linear budget constraint, every selected consumption level weakly increases. The set of all such utility functions is denoted by $\mathcal{U}$.

Each individual i is characterized by a non-labor endowment $\omega_i \geq 0$ and a wage rate $a_i > 0$, which together determine gross income $\omega_i + a_i l_i$, where l_i denotes hours worked.

An *income tax schedule* is defined as a continuous, rank-preserving, and non-decreasing function $t : \mathbb{R}_+ \rightarrow \mathbb{R}$, where $t(y)$ represents the tax liability associated with income level y , subject to the constraint that $t(y) \leq y$ for every $y \geq 0$, which ensures that tax liabilities never exceed pre-tax income levels, maintaining economic feasibility.

The monotonicity requirement embodies a weak form of vertical equity: individuals with higher income cannot face lower tax burdens. By design, this requirement explicitly precludes negative marginal tax rates, a concept that has garnered significant attention in the literature on nonlinear income taxation with endogenous income, both in welfarist and non-welfarist frameworks (see, e.g., Saez, 2002; Saez and Stantcheva, 2016; Boadway et al., 2002; Lockwood, 2020; Kanbur et al., 1994; Fleurbaey and Maniquet, 2006, 2007; Maniquet and Neumann, 2021; Henry de Frahan and Maniquet, 2021). Nevertheless, we demonstrate that this monotonicity constraint can be relaxed without compromising our core results. A detailed analysis of the implications of removing the monotonicity assumption is presented in Appendix B (see also the discussion in Section 4.2.4).

Rank preservation means that post-tax income $y - t(y)$ is non-decreasing in gross income y , preventing taxation from reversing income rankings. Negative tax liabilities represent subsidies.

We restrict attention to *piecewise linear* tax schedules, which partition $\mathbb{R}_+$ into finitely many intervals and are linear on each interval. The set of all such schedules is denoted by $\mathcal{T}$.

We restrict attention to income-contingent tax schedules, under which an individual's liability depends on their own gross income, rather than to vector-valued tax rules that condition individual liabilities on the full profile or distribution of incomes. This is the

conventional policy space in nonlinear income-tax models and the one studied in this paper.

The restriction separates the present problem from a richer mechanism-design environment. Under an income-contingent schedule, one taxpayer's reported income does not directly enter another taxpayer's contemporaneous liability. Under a general vector-valued rule, reports and income choices may instead have cross-taxpayer effects and may therefore become strategically interdependent. Moreover, if liabilities depend on the contemporaneous income distribution, final liabilities generally cannot be determined until the relevant aggregate information has been computed. These strategic and administrative issues fall outside the present analysis.

Horizontal equity would also impose restrictions on the admissible class of vector-valued rules. At a minimum, taxpayers with the same gross income in the same income profile should face the same liability. The income-contingent schedules studied here satisfy the stronger property of own-income invariance: taxpayers with the same gross income face the same liability independently of the surrounding income distribution.

An individual i with wage rate $a_i > 0$, non-labor income $\omega_i \geq 0$, and facing an income tax schedule t , chooses labor $l_i \in [0, 1]$ to solve the utility maximization problem

$$\max_{l_i \in [0,1]} u_i(l_i, \omega_i + a_i l_i - t(\omega_i + a_i l_i)),$$

where $\omega_i + a_i l_i - t(\omega_i + a_i l_i)$ is net income.

This problem has at least one solution due to continuity of u_i and compactness of the feasible set.

A *solution function* $l_i^{u_i}(\omega_i, a_i, t)$ solves this problem given ω_i , a_i , and t .

From $l_i^{u_i}$, define gross and net income functions:

$$y_i^{u_i}(\omega_i, a_i, t) = \omega_i + a_i l_i^{u_i}(\omega_i, a_i, t), \quad x_i^{u_i}(\omega_i, a_i, t) = y_i^{u_i}(\omega_i, a_i, t) - t(y_i^{u_i}(\omega_i, a_i, t)).$$

In an economy of $n \in \mathbb{N}$ individuals, net income distributions are given by

$$(x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t)),$$

parameterized by the wage vector $(a_1, \dots, a_n)$, endowment vector $(\omega_1, \dots, \omega_n)$, and tax schedule t . Each $x_i^{u_i}$ corresponds to individual i 's net income.

The income distribution in the absence of taxation is denoted by

$$(x_1^{u_1}(\omega_1, a_1, 0), \dots, x_n^{u_n}(\omega_n, a_n, 0)).$$

This baseline distribution serves as a reference point for assessing the impact of various tax policies on poverty.

4. Poverty-reducing tax schedules

This paper characterizes income tax schedules that guarantee robust poverty reduction across all possible economic environments. Specifically, we identify tax structures that decrease poverty—as measured by the poverty preorder $\succsim_z$ defined in [Section 2](#)—regardless

of the underlying distribution of endowments and wages, or the heterogeneity in individual preferences.

In formal terms, given a poverty preorder $\succsim$, a tax schedule t is $\succsim$ -poverty-reducing if

$$(x_1^{u_1}(\omega_1, a_1, 0), \dots, x_n^{u_n}(\omega_n, a_n, 0)) \succsim (x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t))$$

for all endowment distributions $(\omega_1, \dots, \omega_n) \in \mathbb{R}_+^n$, all wage distributions $(a_1, \dots, a_n) \in \mathbb{R}_{++}^n$, all preference profiles $(u_1, \dots, u_n) \in \mathcal{U}^n$, and all income solution functions $x_1^{u_1}, \dots, x_n^{u_n}$.²

Our main result completely characterizes the tax schedules that reduce poverty uniformly across all possible wage, endowment, and preference distributions—the set of $\succsim_z$ -poverty-reducing schedules.

Theorem 1. *A tax schedule $t \in \mathcal{T}$ is $\succsim_z$ -poverty-reducing if and only if, letting $s = -t(0)$, one has $s \geq 0$ and $t(y) = -s$ for every $y \in \mathbb{R}_+$ satisfying $y + s \leq z$.*

The subsidy parameter is uniquely determined by the schedule as $s = -t(0)$. When $s < z$, poverty reduction requires this uniform subsidy at every gross-income level y for which $y + s \leq z$. When $s \geq z$, rank preservation implies $y - t(y) \geq -t(0) = s \geq z$ for every $y \geq 0$, so the censored income distribution attains its maximal value.

The formal proof of [Theorem 1](#) appears in [Appendix A](#), with supporting intuition in the next subsection. [Section 4.2](#) discusses the scope and implications of [Theorem 1](#).

4.1. Informal proof of [Theorem 1](#)

To provide intuition for the underlying logic, we offer a visual heuristic argument based on [Figures 1 to 3](#).

Step 1 (Sufficiency): Fixed subsidies below z are poverty-reducing. Suppose, letting $s = -t(0)$, that $s \geq 0$ and $t(y) = -s$ whenever $y + s \leq z$. If $s \geq z$, rank preservation gives $y - t(y) \geq s \geq z$ for every $y \geq 0$, so the result is immediate. Suppose therefore that $s < z$.

Fix arbitrary endowments, wages, preferences, and income solution functions. For each individual i , let

$$c_i = \min\{x_i^{u_i}(\omega_i, a_i, 0), z\} \quad \text{and} \quad c'_i = \min\{x_i^{u_i}(\omega_i, a_i, t), z\}$$

denote tax-free and post-tax income censored at z , respectively. Write

$$c = (c_1, \dots, c_n) \quad \text{and} \quad c' = (c'_1, \dots, c'_n).$$

To establish

$$(x_1^{u_1}(\omega_1, a_1, 0), \dots, x_n^{u_n}(\omega_n, a_n, 0)) \succsim_z (x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t)),$$

it suffices, by [Lemma 1](#), to show that $c'_i \geq c_i$ for every i .

Fix an individual i . As illustrated in [Figure 1](#), on the gross-income range $y + s \leq z$, the schedule acts as a pure subsidy and shifts the budget line upward by s . The formal proof shows that whenever the selected post-tax income remains below z , the taxed optimum

²In the absence of taxation, the solution functions are uniquely determined. However, multiple optimal solutions can emerge under taxation.

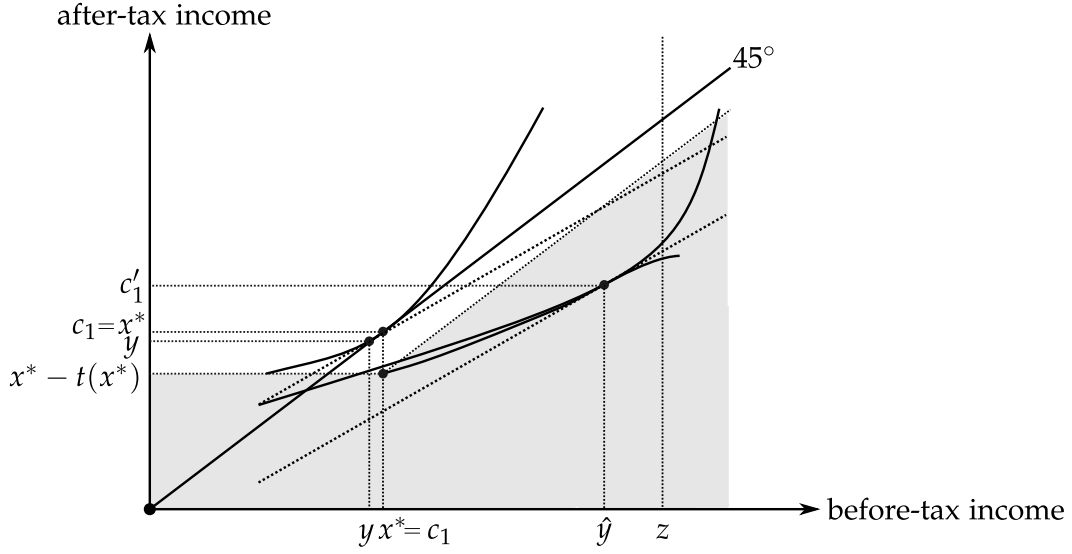


Figure 2: Individual 1's choice problem when $t(x^*) > 0$.

The figure depicts a post-tax choice consistent with $c'_1 \geq c_1$. Relative to the tax-free choice, however, obtaining such an outcome from the depicted budget change would require selected consumption not to respond normally to the corresponding shift in the linear budget comparison.

Thus, $t(x^*) > 0$ is incompatible with universal poverty reduction over the maintained domain of preferences. The formal argument in [Lemma 2](#) makes this contradiction explicit by constructing an admissible preference for which gross income x^* is uniquely optimal under both the tax-free and post-tax schedules, while post-tax consumption is strictly lower. Consequently, $t(y) \leq 0$ for every $y \in [0, z]$.

Step 3 (Necessity II): Subsidies below z must be uniform. We now complete the necessity argument. Suppose that t weakly subsidizes the poor but does not satisfy $t(y) = -s$ for all y with $y + s \leq z$.

Since t weakly subsidizes the poor, part of the budget constraint under t must lie above the 45° line with slope less than unity, as illustrated in [Figure 3](#) by the segment through (y, c'_1) . Consider preferences satisfying consumption normality, as shown in [Figure 3](#). Without taxation, the optimal income is $x^u(0, a, 0) = c_1$, while under t , the optimal after-tax income is $x^u(0, a, t) = c'_1 < c_1$. Constructing homogeneous distributions with these optimal choices, where $z \geq c_1 > c'_1$, yields

$$(x^u(0, a, 0), \dots, x^u(0, a, 0)) \not\succ_z (x^u(0, a, t), \dots, x^u(0, a, t)),$$

contradicting that t is $\succ_z$ -poverty-reducing. This illustrates why a uniform subsidy is necessary for all incomes remaining below z post-subsidy.

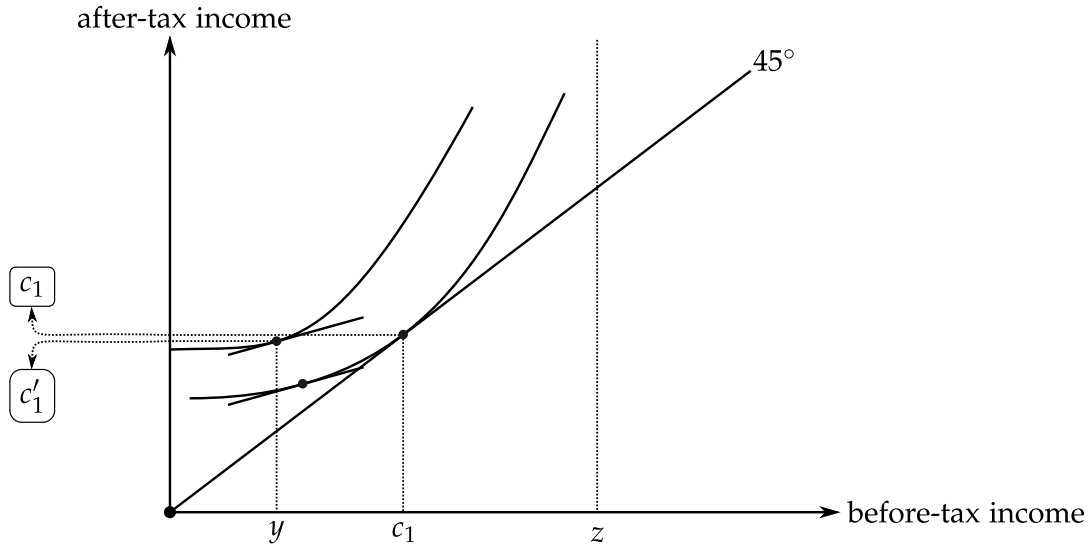


Figure 3: Individual 1's problem.

4.2. Scope, extensions, and robustness

This section examines the scope, economic implications, and robustness of our characterization result ([Theorem 1](#)). The analysis proceeds along several dimensions.

We begin by discussing the context-independence and preference-robustness of uniform lump-sum subsidies. They are independent of resource constraints and robust across admissible preference and endowment heterogeneity. We then show that the set of all poverty-reducing tax schedules is completely ordered by the poverty-relevant subsidy $\sigma_z(t) = \min\{-t(0), z\}$, and that the characterization is invariant across all admissible poverty measures satisfying Focus, Symmetry, Continuity, and Monotonicity.

We next extend the analysis by dropping the requirement that tax liability be non-decreasing. This extension permits negative marginal tax rates. In the enlarged policy space, poverty-reducing schedules must have non-positive tax liabilities at gross incomes weakly below the poverty line and non-positive marginal tax rates over the poverty-relevant income range.

We then examine alternative poverty criteria that incorporate inequality among the poor and discuss the role of rank preservation in the analysis.

The next two extensions address institutional and informational complexities. We consider schedules required to reduce poverty at multiple poverty thresholds and then allow the poverty line to depend on predetermined reference fundamentals.

We also compare our outcome-based framework with the responsibility-sensitive approach of [Henry de Frahan and Maniquet \(2021\)](#) and the poverty-sensitive welfarist approach of [Maniquet and Neumann \(2021\)](#). Despite their different normative foundations, both approaches share with ours a rationale for zero or negative marginal tax rates at low incomes.

We subsequently isolate the role of behavioral responses by comparing endogenous and exogenous income frameworks. With exogenous income, any subsidy to the poor

that preserves non-poor status guarantees universal poverty reduction. When income is endogenous, however, incentive-compatibility constraints restrict the viable policies to uniform subsidies if negative marginal tax rates are excluded.

Finally, we consider two complementary fairness principles. We establish an *equal benefit* result showing that, under quasilinear preferences, uniform subsidies deliver identical utility gains to all agents remaining weakly below the poverty line. Graphical arguments illustrate the roles of income effects and boundary bunching. We also show that, among strictly rank-preserving schedules, *poverty preservation*—the requirement that taxation not invert poverty rankings—implies poverty reduction, while heterogeneous income effects create a potential failure of the converse.

Together, these extensions delineate the boundaries of our main result and clarify how endogenous labor supply reshapes the policy space for tax-mediated poverty alleviation.

4.2.1. Context-independence and preference-robustness

Our analysis notably omits explicit budgetary constraints on admissible tax schedules. Such constraints typically depend on contextual economic factors—specifically, the distribution of wage rates, the resulting gross income distribution, and the government’s resource capacity. Our analysis intentionally leaves these variables unspecified, as our universal poverty reduction framework applies across all economic environments.

Remarkably, this context-independence extends to a broad range of preference domains. The characterization requires only that the admissible domain contain the simple quasilinear functional forms used in the necessity argument, a feature that sharply distinguishes it from related results in inequality-reducing taxation.

To understand this generality, consider the role of homogeneous societies—where all individuals share identical characteristics—in the proof structure. As the informal proof of [Theorem 1 \(Section 4.1\)](#) illustrates, homogeneous societies already impose the complete structure of poverty-reducing tax schedules: uniform subsidies below the poverty line. In such environments, the tax-free ordering of incomes is trivially preserved after taxation, allowing the proof to isolate the essential features of poverty-reducing schedules using only a small collection of standard preferences. The necessity argument in [Appendix A](#) leverages this insight by constructing homogeneous counterexamples using simple quasilinear preferences with strictly convex labor costs, both in [Lemma 2](#) and in the main proof. These counterexamples demonstrate that any tax schedule violating the characterized structure fails to reduce poverty in at least one environment. The theorem therefore requires only that the admissible preference domain contain these standard functional forms.

This domain robustness is absent from the literature on inequality-reducing taxation (see, e.g., [Carbonell-Nicolau, 2025](#)), where homogeneous societies impose no meaningful structure—any tax leaves an already-equal distribution unchanged. There, the characterization must rely on heterogeneous environments where preserving the rank ordering between tax-free and post-tax distributions becomes critical, demanding suitable restrictions on admissible preferences.

4.2.2. Complete preorder of poverty-reducing tax schedules

Beyond preference robustness, the set of universally poverty-reducing tax schedules is completely preordered by a single poverty-relevant parameter. Let

$$\mathcal{T}_{\mathcal{R}_z} = \{t \in \mathcal{T} : \text{letting } s = -t(0), s \geq 0 \text{ and } t(y) = -s \text{ whenever } y + s \leq z\}.$$

For each $t \in \mathcal{T}_{\mathcal{R}_z}$, define its poverty-relevant subsidy by

$$\sigma_z(t) = \min\{-t(0), z\}.$$

The cap reflects censoring: if $-t(0) \geq z$, rank preservation places every attainable net income weakly above z , so larger subsidies at zero income cannot further reduce measured poverty. For any $t, t' \in \mathcal{T}_{\mathcal{R}_z}$,

$$\sigma_z(t) \geq \sigma_z(t') \Leftrightarrow (x_1^{u_1}(\omega_1, a_1, t'), \dots, x_n^{u_n}(\omega_n, a_n, t')) \succsim_z (x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t)) \quad (1)$$

for all endowment distributions, wage distributions, preference profiles, and income solution functions.

To prove the forward implication, fix an individual i and let x^z and x'^z denote the corresponding censored net incomes under t and t' . It suffices to show that $x^z \geq x'^z$.

If $\sigma_z(t) = z$, rank preservation gives $x^z = z$, and the inequality is immediate. Suppose therefore that $\sigma_z(t) < z$. Since $\sigma_z(t') \leq \sigma_z(t) < z$, the cap at z does not bind for either schedule. Hence, writing $s = -t(0)$ and $s' = -t'(0)$, we have $s = \sigma_z(t)$ and $s' = \sigma_z(t')$, and therefore $s \geq s'$.

If $x^z = z$, the result is again immediate. Otherwise, let y be the gross income chosen under t . By [Theorem 1](#),

$$x^z = y - t(y) = y + s < z.$$

In particular, $y < z - s$. The actual budget under t coincides with the pure-subsidy line $\widehat{y} \mapsto \widehat{y} + s$ whenever $\widehat{y} + s \leq z$. Since y is optimal under t , it maximizes the pure-subsidy objective over that initial interval.

The argument used in the sufficiency proof of [Theorem 1](#) then shows that y is the unique solution to the global pure-subsidy problem

$$\max_{\widehat{y} \in [\omega_i, \omega_i + a_i]} u_i \left(\frac{\widehat{y} - \omega_i}{a_i}, \widehat{y} + s \right).$$

Indeed, continuity extends quasiconcavity to the full feasible interval, while strict quasiconcavity need only be applied to intermediate bundles lying in the interior.

Let y' denote the unique solution to the corresponding global pure-subsidy problem with subsidy s' :

$$\max_{\widehat{y} \in [\omega_i, \omega_i + a_i]} u_i \left(\frac{\widehat{y} - \omega_i}{a_i}, \widehat{y} + s' \right).$$

The two pure-subsidy budgets are parallel linear budgets. Since $s \geq s'$, normality of consumption implies $y + s \geq y' + s'$. Because $y + s < z$, it follows that $y' + s' < z$. Hence, by [Theorem 1](#), $t'(y') = -s'$.

Moreover, monotonicity of t' gives $t'(\widehat{y}) \geq t'(0) = -s'$ for every $\widehat{y} \geq 0$, and therefore $\widehat{y} - t'(\widehat{y}) \leq \widehat{y} + s'$. Thus, the actual budget under t' lies weakly below the global pure-subsidy

budget and coincides with it at y' . Since y' is the unique maximizer on the pure-subsidy budget, it is also the unique choice under t' . Consequently,

$$x'^z = y' + s' \leq y + s = x^z.$$

Conversely, suppose $\sigma_z(t) < \sigma_z(t')$. First suppose that $\sigma_z(t') < z$, and write $s = \sigma_z(t)$, $s' = \sigma_z(t')$. Choose a one-agent economy with zero endowment, a wage $a > 0$ satisfying $a + s' < z$, and preferences

$$u(l, x) = x - Kl - \eta l^2,$$

where $\eta > 0$ and $K > a$. Every feasible gross income $y \in [0, a]$ then satisfies $y + s' < z$, so both schedules coincide throughout the feasible interval with their respective pure-subsidy lines. Under either schedule, the utility change from choosing any $l > 0$ instead of $l = 0$ is

$$al - Kl - \eta l^2 < 0.$$

Thus zero labor is uniquely optimal under both schedules. Selected consumption equals s under t and s' under t' . Since $s' > s$, censored income is strictly higher under t' , contradicting the universal ordering on the right-hand side of (1).

If $\sigma_z(t') = z$, schedule t' yields censored income z in every environment, whereas a sufficiently small-wage economy can be chosen so that the outcome under t remains strictly below z . The universal ordering again fails. This proves (1).

4.2.3. Scope: equivalence across poverty measures

To clarify the scope of [Theorem 1](#), we discuss the specific poverty measures it encompasses.

First, recall that for any two income distributions x and y , the relation $x \succsim_z y$ holds if and only if x exhibits no less poverty than y under all poverty measures that satisfy the axioms of Focus, Symmetry, Continuity, and Monotonicity.

This result implies that if a tax schedule t is $\succsim_z$ -poverty-reducing, then it is also poverty-reducing according to *any* poverty measure that satisfies these four axioms.

The converse assertion is also true. To see this, observe that any poverty preorder $\succsim$ satisfying the four axioms is an extension of $\succsim_z$. That is, $\succsim \supseteq \succsim_z$ and $\succ \supseteq \succ_z$.

Now suppose t is $\succsim$ -poverty-reducing. Then, for any endowment distribution $(\omega_1, \dots, \omega_n) \in \mathbb{R}_+^n$, wage distribution $(a_1, \dots, a_n) \in \mathbb{R}_{++}^n$, preference profile $(u_1, \dots, u_n) \in \mathcal{U}^n$, and income solution functions $x_1^{u_1}, \dots, x_n^{u_n}$, we have

$$(x_1^{u_1}(\omega_1, a_1, 0), \dots, x_n^{u_n}(\omega_n, a_n, 0)) \succsim (x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t)).$$

Fix $(\omega_1, \dots, \omega_n)$, $(a_1, \dots, a_n)$, $(u_1, \dots, u_n)$, and $x_1^{u_1}, \dots, x_n^{u_n}$. Then, for each individual i ,

$$(x_i^{u_i}(\omega_i, a_i, 0), \dots, x_i^{u_i}(\omega_i, a_i, 0)) \succsim (x_i^{u_i}(\omega_i, a_i, t), \dots, x_i^{u_i}(\omega_i, a_i, t)). \quad (2)$$

This implies

$$\min\{x_i^{u_i}(\omega_i, a_i, t), z\} \geq \min\{x_i^{u_i}(\omega_i, a_i, 0), z\}. \quad (3)$$

If this inequality failed, we would have

$$(x_i^{u_i}(\omega_i, a_i, t), \dots, x_i^{u_i}(\omega_i, a_i, t)) \succ_z (x_i^{u_i}(\omega_i, a_i, 0), \dots, x_i^{u_i}(\omega_i, a_i, 0)),$$

which would further imply

$$(x_i^{u_i}(\omega_i, a_i, t), \dots, x_i^{u_i}(\omega_i, a_i, t)) > (x_i^{u_i}(\omega_i, a_i, 0), \dots, x_i^{u_i}(\omega_i, a_i, 0))$$

(since $\succsim$ is an extension of $\succsim_z$), contradicting (2).

Since (3) holds for every i , [Lemma 1](#) applied to the censored coordinate vectors implies

$$(x_1^{u_1}(\omega_1, a_1, 0), \dots, x_n^{u_n}(\omega_n, a_n, 0)) \succsim_z (x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t)).$$

Thus, t is also $\succsim_z$ -poverty-reducing.

These insights are formalized in the following corollary to [Theorem 1](#):

Corollary 1 (to [Theorem 1](#)). *Suppose $\succsim$ is an extension of the core preorder $\succsim_z$, i.e., $\succsim \supseteq \succsim_z$ and $> \supseteq >_z$. A tax schedule $t \in \mathcal{T}$ is $\succsim$ -poverty-reducing if and only if, letting $s = -t(0)$, one has $s \geq 0$ and $t(y) = -s$ for every $y \in \mathbb{R}_+$ satisfying $y + s \leq z$.*

The class of poverty measures covered by this corollary includes most well-known indices used in applied and theoretical work. In particular, standard members of the Foster–Greer–Thorbecke (FGT) family ([Foster et al., 1984](#)) with parameter $\alpha > 0$, the Atkinson-style poverty indices ([Atkinson, 1987](#)), the Watts index ([Watts, 1968](#)), and the Sen–Shorrocks–Thon ([Thon, 1979](#); [Shorrocks, 1995](#)) family all satisfy the axioms of Focus, Symmetry, Continuity, and Monotonicity. Consequently, each of these measures induces a poverty ordering that extends the core preorder $\succsim_z$.

Thus, for all such indices, the class of universally poverty-reducing tax schedules is the same.

It is useful to distinguish this universal policy result from the ranking of a particular pair of distributions by a specific index. Some poverty indices, including certain members of the FGT family, may regard as poverty-reducing a change that lifts many individuals to the poverty line while slightly reducing the income of the poorest. The core preorder $\succsim_z$ may instead leave the two distributions incomparable.

This does not affect [Theorem 1](#) or [Corollary 1](#), because poverty reduction is required in every economic environment. If a schedule reduces the income of some poor type in one environment, one can construct a homogeneous economy populated entirely by that type, in which the schedule increases poverty under every admissible measure. The universal requirement therefore excludes policies that trade losses for some poor individuals against gains for others.

4.2.4. Extension to non-monotonic tax schedules

In [Appendix B](#), we characterize poverty-reducing tax schedules when the monotonicity constraint is relaxed, permitting negative marginal tax rates. This extension is particularly relevant given findings in the optimal taxation literature that negative marginal rates can be optimal for low-income individuals (see, e.g., [Kanbur et al., 1994](#); [Maniquet and Neumann, 2021](#); [Henry de Frahan and Maniquet, 2021](#)).

For this extension, we redefine a tax schedule as a continuous, piecewise linear, and rank-preserving function $t : \mathbb{R}_+ \rightarrow \mathbb{R}$ satisfying $t(y) \leq y$ for all $y \geq 0$, dropping the monotonicity requirement. Let $\mathcal{T}^*$ denote this expanded set of admissible schedules.

Theorem 2. For $t \in \mathcal{T}^*$, let

$$S_z(t) = \{y \in [0, z] : y - t(y) < z\}.$$

A tax schedule $t \in \mathcal{T}^*$ is $\succsim_z$ -poverty-reducing if and only if $t(y) \leq 0$ for every $y \in [0, z]$ and t is non-increasing on $S_z(t)$.

The formal proof is provided in [Appendix B](#). While pure subsidies ([Theorem 1](#)) are no longer the unique solution, two core insights remain: (i) tax liabilities must be non-positive at gross incomes weakly below the poverty line, and (ii) positive marginal tax rates are incompatible with poverty reduction on the poverty-relevant region $S_z(t)$ —equivalently, where gross income is at most z and net income remains strictly below z . The strict inequality in the definition of $S_z(t)$ is essential: once net income reaches z , changes that leave censored income equal to z do not affect measured poverty. Negative marginal tax rates remain permissible as a means of strengthening work incentives while subsidizing the poor.

4.2.5. Alternative poverty criteria

The main analysis employs the censored first-order stochastic dominance (FOSD) criterion $\succsim_z$. However, recent work has developed distribution-sensitive poverty measures that explicitly account for inequality among the poor. Following [Carbonell-Nicolau \(2026\)](#), we consider two such criteria: the relative poverty preorder $\succsim_R^z$ and its absolute counterpart $\succsim_A^z$. These measures satisfy the standard axioms of Focus, Symmetry, and Continuity, but replace the Monotonicity axiom with distribution-sensitivity axioms. Specifically, they combine the Minimal Transfer axiom with either Relative Weak Monotonicity (for $\succsim_R^z$) or Absolute Weak Monotonicity (for $\succsim_A^z$), as detailed in [Carbonell-Nicolau \(2026\)](#).

Intuitively, these criteria evaluate poverty through inequality-adjusted measures introduced by [Shorrocks \(1983\)](#), applied to censored income distributions. Under $\succsim_R^z$, a distribution y exhibits less relative poverty than x if the censored distribution y^z is at least as equal as x^z in relative terms (via the relative Lorenz criterion) *and* has weakly higher total income. Analogously, $\succsim_A^z$ applies the absolute Lorenz criterion to censored distributions, requiring that absolute income gaps among the poor do not widen while total income does not decline. Both criteria need not rank income increases as poverty-reducing when they exacerbate inequality among the poor, even if every poor individual becomes richer in absolute terms. In contrast, censored FOSD would deem such changes unambiguously poverty-reducing. As demonstrated in [Carbonell-Nicolau \(2026\)](#), these criteria are non-nested with censored FOSD: neither implies the other in general.

A natural question is whether [Theorem 1](#) extends to these distribution-sensitive poverty criteria. The answer is partial. The necessity direction carries over completely:

Theorem 3. If a tax schedule $t \in \mathcal{T}$ is $\succsim_R^z$ -poverty-reducing (respectively, $\succsim_A^z$ -poverty-reducing), then, letting $s = -t(0)$, one has $s \geq 0$ and $t(y) = -s$ for every $y \in \mathbb{R}_+$ satisfying $y + s \leq z$.

This result follows directly from the formal necessity proof of [Theorem 1](#). Whenever the fixed-subsidy condition fails, that proof constructs a homogeneous economy in which every individual has the same tax-free income and the same strictly lower post-tax income, with both incomes below z . The corresponding censored distributions are constant vectors. Their inequality components are therefore identical, while total censored income

is strictly lower under taxation. Consequently, $\succ_R^z$ and $\succ_A^z$, as well as $\succ_z$, strictly rank the tax-free distribution above the post-tax distribution. Hence any tax schedule violating the fixed-subsidy condition fails to reduce poverty under either alternative criterion.

The sufficiency direction need not extend to $\succ_R^z$ or $\succ_A^z$. The following behavioral configuration illustrates the obstruction. Consider a fixed lump-sum subsidy $t(y) = -s$ for $y + s \leq z$, with $s > 0$, applied to a two-agent economy where both agents remain poor before and after taxation. Suppose preferences or wage rates are such that the subsidy induces a 10% consumption increase for the poorer agent but a 15% increase for the richer poor agent.

Such heterogeneous responses are entirely plausible under our framework, which allows arbitrary preference heterogeneity (and can even arise with common preferences given sufficient wage dispersion). While both agents become absolutely better off, relative inequality among the poor increases. Under $\succ_R^z$, the post-tax censored distribution neither dominates nor is dominated by the tax-free distribution. An analogous configuration involving unequal absolute, rather than proportional, consumption gains illustrates the same incomparability under $\succ_A^z$.

This configuration illustrates why the sufficiency argument for censored FOSD does not automatically extend to $\succ_R^z$ or $\succ_A^z$ across fully heterogeneous preference profiles. However, the literature on tax-mediated inequality and polarization (see, e.g., [Carbonell-Nicolau and Llavador, 2018, 2021a,b, 2025](#)) suggests that under the common assumption of homogeneous preferences, it is possible to derive conditions on wage elasticities such that uniform lump-sum subsidies (which are marginal-rate progressive) are poverty-reducing with respect to $\succ_R^z$ and $\succ_A^z$.

4.2.6. On the role of rank preservation

The rank-preservation requirement ensures that higher gross income never yields lower net income. Any violation creates dominated choices for individuals who can attain the relevant lower gross-income level. Specifically, if $y' < y$ and $y' - t(y') \geq y - t(y)$, then an individual with endowment $\omega \leq y'$ obtains at least as much consumption and supplies less labor by choosing y' rather than y . Hence y cannot be optimal for that individual. The dominance argument is type-dependent, however. If $\omega > y'$, the lower gross-income choice y' is infeasible and does not dominate y within the individual's attainable set. The behaviorally relevant upper envelope is therefore

$$\tilde{x}_\omega(y) = \max_{\omega \leq y' \leq y} \{y' - t(y')\},$$

which depends on the individual's endowment. Consequently, the behavioral irrelevance of dominated portions of a schedule does not by itself yield a single, type-independent replacement for a rank-violating schedule. We therefore retain rank preservation as part of the admissible policy space.

Rank preservation also serves economic purposes beyond analytical convenience. Normatively, it embodies a minimal horizontal equity principle—taxation should not reverse income rankings—widely regarded as a baseline fairness requirement. Practically, a violation would create perverse incentives for workers to reduce output, generating inefficient bunching at kink points. Analytically, rank preservation avoids the need to

replace each schedule with an endowment-dependent upper envelope before analyzing individual choice.

4.2.7. Robustness across multiple poverty lines

The characterization in [Theorem 1](#) assumes a fixed poverty line. Because the appropriate threshold may be contested, we now require a single tax schedule to reduce poverty across a collection of possible poverty lines. Let $Z \subseteq \mathbb{R}_{++}$ be nonempty and bounded above. The schedule must be $\succsim_z$ -poverty-reducing for every $z \in Z$.

Let $s = -t(0)$. If $s < 0$, the schedule fails the fixed-threshold characterization for every $z \in Z$. Suppose $s \geq 0$. If $t(y) = -s$ whenever $y + s \leq \sup Z$, then for every $z \in Z$ the same equality holds whenever $y + s \leq z$, and [Theorem 1](#) implies poverty reduction at z .

Conversely, suppose t is poverty-reducing for every $z \in Z$. By [Theorem 1](#), the subsidy parameter for each threshold equals the same number $s = -t(0) \geq 0$. If $y + s < \sup Z$, choose $z \in Z$ with $y + s \leq z$; then [Theorem 1](#) gives $t(y) = -s$. If $y + s = \sup Z$ and $y > 0$, approximate y from below and use continuity of t . If $y = 0$, the conclusion follows directly from $t(0) = -s$. Thus $t(y) = -s$ whenever $y + s \leq \sup Z$. When $s \geq \sup Z$, the condition is automatic at the boundary and rank preservation gives $y - t(y) \geq s \geq z$ for every $z \in Z$.

Corollary 2 (to [Theorem 1](#)). *Let $Z \subseteq \mathbb{R}_{++}$ be nonempty and bounded above. A tax schedule $t \in \mathcal{T}$ is $\succsim_z$ -poverty-reducing for every $z \in Z$ if and only if, letting $s = -t(0)$, one has $s \geq 0$ and $t(y) = -s$ for every $y \in \mathbb{R}_+$ satisfying $y + s \leq \sup Z$.*

4.2.8. Endogenous poverty lines

Throughout the paper the poverty line $z > 0$ is treated as exogenously given. In practice, however, many official thresholds are defined relative to the income distribution itself—for example as a fixed percentage of median income. This raises the question of whether [Theorem 1](#) extends to such endogenous poverty lines.

The answer depends on how the poverty line is determined. When the threshold is defined using a *post-tax* statistic (e.g., a fraction of the post-tax median), a circularity problem arises: the poverty line z is itself an equilibrium object that depends on the tax schedule t under consideration. Evaluating policy relative to z therefore requires solving a fixed-point problem (t, z) , in which each variable depends on the other. This circularity constitutes a fundamental obstacle that we do not address in this paper.

Circularity can be avoided by defining the poverty line using a *predetermined* statistic—such as a lagged median or any function of structural primitives. In such cases, the threshold becomes a well-defined mapping $\zeta = \zeta(\omega', a', u')$, where (ω', a', u') serve as reference parameters that fix the poverty line prior to policy implementation (e.g., last year's economic fundamentals).³

This raises a natural question: does there exist a single tax schedule t satisfying

$$(x_1^{u_1}(\omega_1, a_1, 0), \dots, x_n^{u_n}(\omega_n, a_n, 0)) \succsim_{\zeta(\omega', a', u')} (x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t))$$

³For comparison, in environments with *exogenous* incomes, [Carbonell-Nicolau \(2026\)](#) handles endogenous poverty lines via normalization arguments (exploiting scale or translation invariance). Such approaches fail here because behavioral responses break scaling equivariance: with endogenous labor supply, proportional changes in primitives generally induce non-proportional income adjustments. We therefore adopt a direct characterization rather than pursuing a normalization strategy.

for *all* reference parameters (ω', a', u') and all actual economies (ω, a, u) ?

The answer is affirmative. Let

$$Z = \{\zeta(\omega', a', u') : (\omega', a', u') \in \mathbb{R}_+^n \times \mathbb{R}_{++}^n \times \mathcal{U}^n\}$$

denote the range of the poverty-line function and suppose that $\sup Z < +\infty$. By [Corollary 2](#), a single schedule t is poverty-reducing for every threshold in Z if and only if, letting $s = -t(0)$, one has $s \geq 0$ and $t(y) = -s$ whenever $y + s \leq \sup Z$. If $s \leq \sup Z$, the subsidy must remain uniform through gross income $\sup Z - s$; if $s \geq \sup Z$, rank preservation places every attainable net income weakly above every threshold in Z .

Corollary 3 (to [Theorem 1](#)). *Let $\zeta : \mathbb{R}_+^n \times \mathbb{R}_{++}^n \times \mathcal{U}^n \rightarrow \mathbb{R}_{++}$ be a poverty-line function, let Z denote its range, and suppose that $\sup Z < +\infty$. A tax schedule $t \in \mathcal{T}$ is $\succsim_z$ -poverty-reducing for every $z \in Z$ if and only if, letting $s = -t(0)$, one has $s \geq 0$ and $t(y) = -s$ for every $y \in \mathbb{R}_+$ satisfying $y + s \leq \sup Z$.*

This characterization upholds the robustness requirement central to [Theorem 1](#): poverty-reducing policies must work across all economic environments, not merely specific configurations of fundamentals. Even when calibrated to reference information—such as lagged income distributions—the actual implementation environment (wages, endowments, preferences) remains uncertain. The single-schedule solution in [Corollary 3](#) satisfies this requirement without requiring policymakers to adjust the tax code as reference statistics evolve.

4.2.9. Welfarist and responsibility-sensitive approaches versus outcome-based poverty reduction

The responsibility-sensitive framework, as formalized by [Fleurbaey and Maniquet \(2006, 2007\)](#) and recently applied to poverty reduction by [Henry de Frahan and Maniquet \(2021\)](#), distinguishes between inequalities arising from circumstances beyond individual control (e.g., wage rates) and those reflecting autonomous preference differences (e.g., taste for leisure). Their social welfare functions satisfy an equal-wage transfer axiom: absent informational constraints, no redistribution should occur among agents with identical wages but different labor supplies. This principle implies that deserved poverty—resulting from freely chosen low labor supply among agents with adequate earning capacity—should not be remedied through redistribution, whereas undeserved poverty stemming from low wages warrants intervention. Their optimal tax formula features zero or negative marginal rates on low incomes, followed by a discrete jump to positive rates just above the income level of the hardworking poor (those working full time at the minimum wage), with bunching at this threshold.

A related but distinct welfarist approach is developed by [Maniquet and Neumann \(2021\)](#). They combine a poverty-reduction principle with Pareto efficiency and robustness requirements. Their poverty-reduction principle favors a transfer from an individual above the poverty line to an individual below it when the two supply the same amount of labor. The equal-labor condition makes the comparison sensitive to individual well-being, since the individual with lower consumption is then unambiguously worse off. The resulting social ordering leads to a policy criterion that minimizes the labor time a minimum-wage worker must supply to attain the poverty line. Their analysis also provides a rationale

for zero or negative marginal tax rates at low incomes, although it rests on a different normative foundation from the responsibility-sensitive approach.

By contrast, our framework adopts a public-reason perspective in which income below a socially recognized threshold constitutes an objective deprivation demanding redress irrespective of its behavioral origin. This normative stance reflects taxpayers' concern with observable manifestations of poverty rather than unobservable leisure choices (Besley and Coate, 1995), and yields a uniform lump-sum subsidy for all individuals whose post-tax incomes fall below the poverty line—a policy robust across all preference profiles and economic environments.

Despite these normative differences, both frameworks converge on rejecting positive marginal tax rates over the poverty-relevant income range. In our setting, such rates violate universal poverty reduction through adverse substitution effects when net income remains strictly below the poverty line. In the responsibility framework, they violate fairness by transferring resources from hardworking to less hardworking agents with identical wages.

4.2.10. Endogenous versus exogenous income

To isolate the role of behavioral responses in poverty-reducing taxation, we compare the endogenous income framework developed in this paper with its exogenous counterpart. This comparison requires adopting a common poverty criterion. While the exogenous income analysis in Carbonell-Nicolau (2026) employs distribution-sensitive poverty measures non-nested with censored FOSD, we can adapt those methods to characterize poverty-reducing taxes under censored FOSD in the exogenous case, yielding a clean benchmark for comparison.

In the exogenous setting, a *tax schedule* is a continuous function $t: \mathbb{R}_+ \rightarrow \mathbb{R}$ satisfying $t(x) \leq x$ for all $x \geq 0$.

Given a poverty line $z > 0$, a tax schedule t is $\succsim_z$ -poverty-reducing if, for every pre-tax income distribution $\mathbf{x} = (x_1, \dots, x_n)$,

$$\mathbf{x} \succsim_z (x_1 - t(x_1), \dots, x_n - t(x_n)).$$

The following result characterizes poverty-reducing tax schedules when income is exogenous:

Proposition 1. *A tax schedule t is $\succsim_z$ -poverty-reducing if and only if it satisfies:*

- (i) *Non-poor status preservation:* $x > z \Rightarrow x - t(x) \geq z$;
- (ii) *Weak subsidization:* $t(x) \leq 0$ for all $x \in [0, z]$.

Condition (i) ensures that taxation does not push the initially non-poor into poverty—a basic fairness requirement. Condition (ii) guarantees that every poor individual receives a non-negative transfer.

The characterization in Proposition 1 stands in contrast to the findings in Carbonell-Nicolau (2026), where poverty-reducing tax schedules relative to distribution-sensitive poverty measures require additional structure: rank preservation among the poor and average-rate progressivity.

Proof of Proposition 1. Sufficiency. Suppose t satisfies (i) and (ii). Let x be any pre-tax distribution and

$$y = (x_1 - t(x_1), \dots, x_n - t(x_n))$$

the post-tax distribution. Censor both at z to obtain x^z and y^z . For each individual i :

- If $x_i \leq z$, then $y_i = x_i - t(x_i) \geq x_i$ by (ii), so $y_i^z \geq x_i^z$.
- If $x_i > z$, then $y_i \geq z$ by (i), so $y_i^z = z = x_i^z$.

Thus $y_i^z \geq x_i^z$ for all i , which implies $x \succsim_z y$.

Necessity. Suppose first that $t(x^*) > 0$ for some $x^* \in [0, z]$. Consider the homogeneous distribution $x = (x^*, \dots, x^*)$. Then

$$y = (x^* - t(x^*), \dots, x^* - t(x^*))$$

with $y_i < x_i \leq z$, so $y^z = y$ and $x^z = x$. Since $y_i < x_i$ for all i , we have $x \not\succsim_z y$, contradicting poverty reduction.

Second, suppose t violates non-poor status preservation: there exists $x' > z$ with $x' - t(x') < z$. Consider $x = (x', \dots, x')$. Then $x^z = (z, \dots, z)$ but

$$y^z = (x' - t(x'), \dots, x' - t(x')),$$

with all components less than z , so again $x \not\succsim_z y$.

Hence both (i) and (ii) are necessary. ■

Under $\succsim_z$, only individual income levels matter—not their relative ordering or dispersion. This allows for tax schedules that violate rank preservation or monotonicity yet still reduce poverty.

Example 1 (rank reversal without increasing poverty). Let $z = 10$, and consider two poor individuals with pre-tax incomes (4, 6). Define a tax schedule with subsidies $t(4) = -3$ and $t(6) = -1$, so post-tax incomes are (7, 5). Ranks are reversed, but both individuals are better off. Thus poverty is reduced despite rank reversal.

Example 2 (non-monotonic subsidies). Let $z = 10$, and consider two individuals with pre-tax incomes (3, 5). Define a tax schedule with $t(3) = -1$ and $t(5) = -3$, so that tax liability decreases as income increases—violating monotonicity. Nevertheless, post-tax incomes are (4, 8), which dominate the pre-tax vector (3, 5) pointwise.

The divergence between Proposition 1 and Theorem 1 is driven by behavioral responses. With exogenous income, any non-positive tax on $[0, z]$ that preserves non-poor status raises income. With endogenous income and nonnegative marginal tax rates, however, declining subsidies may cause individuals to work less, with post-subsidy income falling below its tax-free level.

When marginal tax rates are restricted to be nonnegative, only a uniform lump-sum subsidy eliminates the substitution effect and guarantees universal poverty reduction. Its zero marginal tax rates generate a pure income effect, raising consumption for all poor individuals. When negative marginal tax rates are admitted, poverty reduction may instead be supported by pro-work substitution effects, as shown in Theorem 2.

4.2.11. Equal benefit and poverty-reducing subsidies

We adapt Young's (1987; 1988; 1990) equal sacrifice principle to the context of poverty-reducing subsidies. While Young's framework focuses on the equitable distribution of utility *losses* through taxation, our setting considers utility *gains* accruing from transfers to the poor. Restricting attention to the family of poverty-reducing schedules characterized in [Theorem 1](#), we ask which policies deliver *equal benefit* to those who remain poor after subsidization.

This criterion stipulates that any individual remaining weakly below the poverty line post-subsidy must experience an identical welfare improvement. We demonstrate that poverty-reducing schedules satisfy equal benefit if preferences are quasilinear. We then use graphical arguments to explain how a kink at $z - s$ and income effects on labor supply can make utility gains vary across types.

Definition 1. A tax schedule $t \in \mathcal{T}$ satisfies *equal benefit among the poor with respect to* $u \in \mathcal{U}$ if there exists a constant $\alpha \geq 0$ such that

$$u(l^u(\omega, a, t), x^u(\omega, a, t)) - u(l^u(\omega, a, 0), x^u(\omega, a, 0)) = \alpha$$

for all (ω, a) such that $x^u(\omega, a, t) \leq z$.

Because this criterion compares utility differences, equal benefit is understood relative to the maintained cardinal normalization of the utility representation.

A utility function $u \in \mathcal{U}$ is *quasilinear* if it takes the form $u(l, x) = x - v(l)$, where $v : [0, 1] \rightarrow \mathbb{R}$ is continuous, strictly increasing, strictly convex, and differentiable on $(0, 1)$. Under quasilinearity, the marginal utility of consumption is constant, eliminating income effects on labor supply while maintaining normality of consumption.

The analysis below concerns the nondegenerate case $s < z$. When $s > z$, no individual remains poor. When $s = z$, only individuals attaining net income exactly z can satisfy the maintained definition of poverty. For $s < z$, [Theorem 1](#) requires $t_s(y) = -s$ whenever $y + s \leq z$. The following proposition identifies conditions under which such schedules implement equal benefit among the poor.

Proposition 2. A member t_s of the family of $\succsim_z$ -poverty-reducing tax schedules implements equal benefit among the poor with respect to $u \in \mathcal{U}$ if u is quasilinear and t_s has no kink point at $z - s$.

The intuition is straightforward: equal benefit requires that all agents remaining poor post-subsidy experience an identical utility gain. Quasilinear preferences eliminate income effects on labor supply and make utility affine in consumption. The proof below shows that an individual remaining weakly below the poverty line globally faces the same labor-supply comparison as under a pure lump-sum subsidy. Consequently, labor supply is unchanged and the utility gain equals s . The no-kink condition at $z - s$ ensures that the subsidy remains uniform throughout the domain of agents who stay poor.

Proof of Proposition 2. Fix a type (ω, a) satisfying $x^u(\omega, a, t_s) \leq z$, and let $y' = y^u(\omega, a, t_s)$. Since $t_s(y) = -s$ for $y \leq z - s$ and t_s has no kink at $z - s$, the schedule remains equal to $-s$ on a right neighborhood of $z - s$. Rank preservation then implies that net income is strictly greater than z whenever $y > z - s$. Hence

$$y' \leq z - s \quad \text{and} \quad x^u(\omega, a, t_s) = y' + s.$$

Monotonicity of t_s and $t_s(0) = -s$ imply

$$y - t_s(y) \leq y + s, \quad \text{for every } y \geq 0.$$

Thus the actual budget lies weakly below the global pure-subsidy line and coincides with it on an interval containing $[0, z - s]$ and a right neighborhood of $z - s$.

Because y' is optimal under t_s , it maximizes the pure-subsidy objective

$$-v\left(\frac{y - \omega}{a}\right) + y + s$$

over that initial interval. This objective is strictly concave. If a feasible point farther to the right yielded weakly greater utility, an intermediate point in the interval on which the two budgets coincide would yield strictly greater utility than y' , contradicting optimality under t_s . Hence y' is the unique global maximizer of the pure-subsidy objective.

The pure-subsidy objective differs from the tax-free objective

$$-v\left(\frac{y - \omega}{a}\right) + y$$

only by the constant s . Their unique maximizers therefore coincide, so

$$l^u(\omega, a, t_s) = l^u(\omega, a, 0).$$

Consequently,

$$u(l^u(\omega, a, t_s), x^u(\omega, a, t_s)) - u(l^u(\omega, a, 0), x^u(\omega, a, 0)) = s.$$

The utility gain is therefore the same for every type remaining weakly below z , so t_s implements equal benefit with $\alpha = s$. ■

Obstacles to equal benefit. The sufficient result in [Proposition 2](#) relies on two economically distinct features: the absence of a kink at the boundary $z - s$ of the uniform-subsidy region and the absence of income effects on labor supply. The following graphical arguments explain how either feature can make the utility gain from the subsidy vary across types. They are intended to identify the mechanisms behind the sufficient conditions, rather than to provide a formal converse to [Proposition 2](#).

A kink at $z - s$. Suppose that the subsidy begins to phase out immediately above $z - s$, so the net-income schedule has a kink at $(z - s, z)$. As illustrated in [Figure 4](#), some types may bunch at this kink. Consider two nearby wages for which the post-subsidy optimum remains strictly at $(z - s, z)$. Post-subsidy utility is then the same for the two types: both choose zero labor from endowment $z - s$ and consume z . By contrast, if the tax-free choice uses positive labor, a higher wage strictly expands the attainable tax-free consumption possibilities and raises tax-free utility. The utility gain from the subsidy therefore falls with the wage. Accordingly, whenever the kink attracts a locally stable set of types, equal benefit fails.

This argument also clarifies the role of the no-kink condition in [Proposition 2](#). Without a kink at $z - s$, the pure-subsidy segment continues to the right of the poverty boundary. A type remaining weakly poor cannot then be held at that boundary by bunching while its tax-free utility varies with the wage.

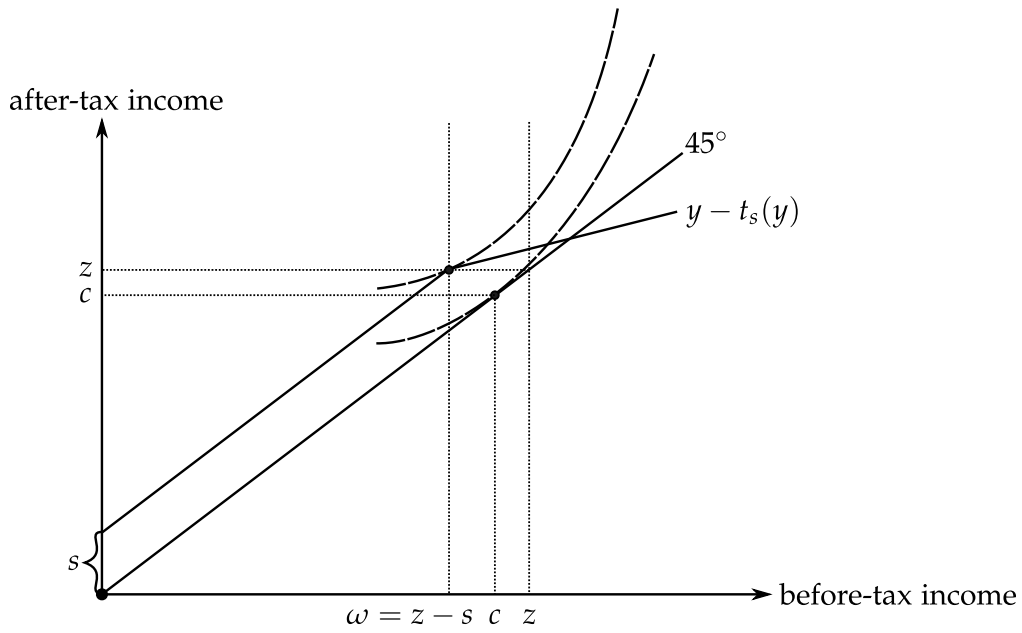


Figure 4: Endowment $\omega = z - s$ and wage a ; tax-free income $c = y^u(\omega, a, 0)$. A wage increase that leaves the post-subsidy choice at the kink raises tax-free utility but not post-subsidy utility.

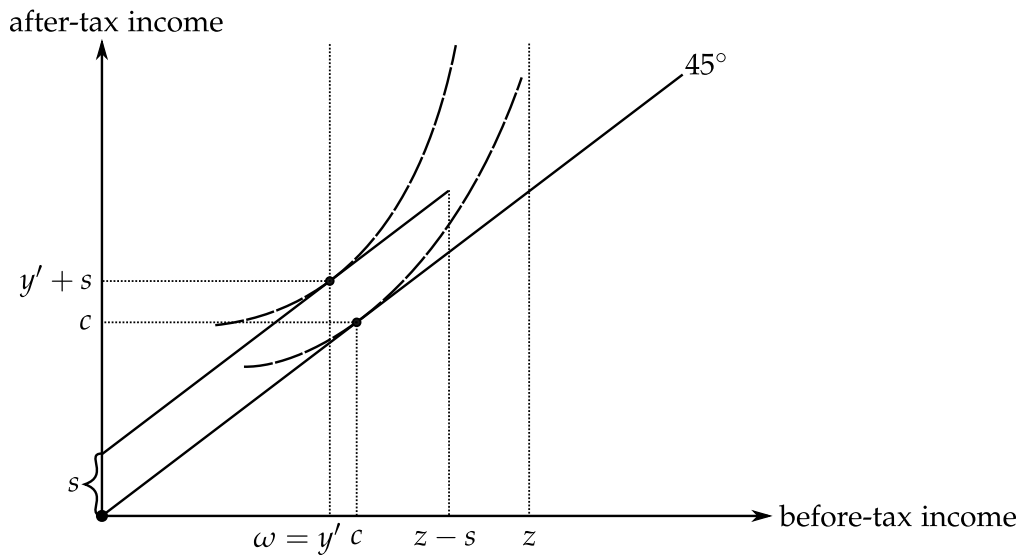


Figure 5: Endowment $\omega = y'$ and wage a' ; tax-free income $c = y^u(y', a', 0) > y'$. A small wage decrease that leaves the post-subsidy choice at $(y', y' + s)$ lowers tax-free utility while leaving post-subsidy utility unchanged.

Income effects. Even in the interior of the uniform-subsidy region, a common monetary transfer need not generate a common utility gain. Fix a post-subsidy bundle $(y', y' + s)$ with $y' \leq z - s$, and consider a type whose non-labor endowment equals y' . The subsidized bundle then entails zero labor. Suppose that removing the subsidy induces the individual to supply positive labor, so that the tax-free choice has gross income strictly greater than y' .

As illustrated in [Figure 5](#), consider a small decrease in the wage that leaves the post-subsidy optimum locally at $(y', y' + s)$. Because this bundle entails zero labor, post-subsidy utility is unchanged. The lower wage, however, contracts the tax-free opportunity set and, because the tax-free choice involves positive labor, lowers tax-free utility. The utility gain from the subsidy therefore increases as the wage falls. Nearby types consequently obtain different utility gains from the same monetary subsidy.

Quasilinear preferences eliminate this channel: adding or removing the lump-sum amount s does not change the labor choice, and utility is affine in consumption. The common monetary subsidy therefore produces the common utility gain s , as established in [Proposition 2](#).

These observations explain why the two assumptions in [Proposition 2](#) are natural. The absence of a kink prevents wage-dependent bunching at the boundary of the uniform-subsidy region, while quasilinearity prevents heterogeneous labor-supply responses from making the utility gain type-specific. The graphical arguments suggest that equal benefit cannot generally be sustained beyond the quasilinear case.

4.2.12. Poverty preservation

We introduce a fairness principle based on order consistency, analogous to the familiar rank-preservation requirement in standard tax design. Rank preservation within a society requires that taxation not reverse the income ordering of individuals: those who are richer pre-tax should not become poorer post-tax. Our requirement applies the same logic across distributions rather than individuals. If one society exhibits weakly less poverty than another before taxation, then taxation should not reverse that ordering afterward. Poverty preservation is thus the cross-society analogue of rank preservation: it rules out rank reversals in poverty standings across economies.

In the endogenous income framework, this principle must be formalized with respect to the underlying fundamentals (ω, a, u) that generate income distributions. For a tax schedule t , let

$$x^u(\omega, a, t) = (x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t))$$

denote the post-tax income distribution induced by fundamentals (ω, a, u) .

Definition 2. A tax schedule t satisfies *poverty preservation* if for all vector fundamentals (ω, a, u) and (ω', a', u') with

$$x^{u'}(\omega', a', 0) \succ_z x^u(\omega, a, 0),$$

we have

$$x^{u'}(\omega', a', t) \succ_z x^u(\omega, a, t).$$

For this subsection only, call a tax schedule $t \in \mathcal{T}$ *strictly rank-preserving* if the associated net-income schedule $y \mapsto y - t(y)$ is strictly increasing. Because t is piecewise linear with

finitely many brackets, this is equivalent to requiring every marginal tax rate to be strictly below 100%.

Under this additional restriction, poverty preservation implies poverty reduction.

Proposition 3. *Any strictly rank-preserving tax schedule that satisfies poverty preservation is $\succsim_z$ -poverty-reducing.*

Proof. Suppose that a strictly rank-preserving schedule t is not $\succsim_z$ -poverty-reducing. Feasibility gives $t(0) \leq 0$; let $s = -t(0) \geq 0$. By [Theorem 1](#), there exists $y > 0$ such that

$$y + s \leq z \quad \text{and} \quad t(y) > t(0) = -s.$$

Because t is nondecreasing and piecewise linear, some bracket with a strictly positive marginal tax rate contains an interior point $\bar{y} \in (0, y)$. Let $q(r) = r - t(r)$ denote net income. Since $t(\bar{y}) \geq t(0) = -s$,

$$q(\bar{y}) \leq \bar{y} + s < y + s \leq z.$$

Thus $\bar{y}$ lies in a poverty-relevant positive-rate bracket. Strict rank preservation implies that the marginal tax rate on this bracket belongs to $(0, 1)$; denote it by τ .

We construct two one-person economies with the same tax-free income but different post-tax incomes. Because $\bar{y}$ lies in the interior of the selected bracket and $q(\bar{y}) < z$, choose $\omega \geq 0$ and $a > 0$ so that the entire feasible gross-income interval $[\omega, \omega + a]$ lies in the interior of the bracket, satisfies

$$q(r) < z, \quad \text{for every } r \in [\omega, \omega + a],$$

and contains $\bar{y}$ as $\bar{y} = \omega + a\bar{l}$ for some $\bar{l} \in (0, 1)$. For $j \in \{1, 2\}$, consider quasilinear preferences $u_j(l, x) = x - v_j(l)$, where v_j is strictly increasing and strictly convex and is chosen so that $v'_j(\bar{l}) = a$. These preferences belong to $\mathcal{U}$: they are strictly increasing in consumption, strictly decreasing in labor, strictly quasiconcave, and consumption is normal. Under the tax-free schedule, both agents uniquely choose $\bar{l}$, and hence both economies generate the same income $\bar{y}$.

Because the feasible gross-income interval lies within one bracket, the taxed problem is globally linear over the individual's choice set, and its unique solution l_j satisfies

$$a(1 - \tau) = v'_j(l_j).$$

Since $v'_j(\bar{l}) = a$ and $\tau > 0$, strict convexity implies $l_j < \bar{l}$. For example, choose two distinct parameters κ_1, κ_2 satisfying

$$\frac{a\tau}{\bar{l}} < \kappa_j < \frac{a}{\bar{l}}, \quad j \in \{1, 2\},$$

and define

$$v'_j(l) = a + \kappa_j(l - \bar{l}).$$

Then $v'_j(l) > 0$ on $[0, 1]$, so v_j is strictly increasing, and $v''_j(l) = \kappa_j > 0$, so v_j is strictly convex. The unique taxed choice is

$$l_j = \bar{l} - \frac{a\tau}{\kappa_j} \in (0, \bar{l}).$$

Because $\kappa_1 \neq \kappa_2$, one has $l_1 \neq l_2$. Both post-tax incomes therefore lie below z . Strict rank preservation then gives

$$q(\omega + al_1) \neq q(\omega + al_2).$$

The two economies are therefore poverty-equivalent under the tax-free schedule but not under t . Because their tax-free distributions are identical, poverty preservation applies in both directions and requires their post-tax distributions to remain poverty-equivalent. For one-person economies with both post-tax incomes below z , this requires equality of post-tax incomes, contradicting the preceding inequality. Hence a strictly rank-preserving schedule that is not $\succsim_z$ -poverty-reducing cannot satisfy poverty preservation. ■

The converse implication need not be expected to hold. Although uniform lump-sum subsidies satisfy poverty reduction ([Theorem 1](#)), heterogeneous income effects can in principle reverse the poverty ranking of two economies. The following argument illustrates this mechanism.

Consider two societies, each with one poor individual. In the tax-free environment, agent A consumes more than agent B , so society A exhibits strictly less poverty than society B . Now implement a uniform subsidy $s > 0$ that leaves both agents poor post-tax. Because preferences differ, agent A may reduce labor supply more aggressively than agent B in response to the income effect. If this behavioral response is sufficiently heterogeneous, post-tax income ranks reverse—reversing the poverty ranking.

Both agents still experience poverty reduction individually, satisfying the requirement of [Theorem 1](#). Yet the relative poverty positions of the two societies are scrambled: a policy that improves each society internally fails to preserve their pre-tax poverty ordering. This reversal occurs because heterogeneous behavioral responses distort the cross-society comparison.

Thus, poverty reduction does not by itself ensure preservation of poverty rankings across heterogeneous economies. The former demands improvement *within* each society; the latter demands consistency *across* societies. Heterogeneous behavioral responses create a channel through which the ranking may be reversed, a tension that cannot generally be ruled out without restricting the domain of admissible preferences.

5. Concluding remarks

This paper treats poverty reduction as an explicit non-utilitarian policy objective, a criterion that aligns with what [Kanbur et al. \(2006\)](#) identify as “behavioral public economics,” wherein the government optimizes based on objectives distinct from individuals’ utility functions. By shifting toward a concrete and widely recognized policy goal, this approach avoids the fundamental challenge of interpersonal utility comparisons in classical welfare economics. However, since poverty measures are typically income-based, they do not capture the individual disutility of work effort. From a positive perspective, this omission may be justified by the perceptions of taxpayers who finance poverty alleviation programs, as they may place greater emphasis on the income of the poor rather than their leisure choices ([Besley and Coate, 1995](#)).

This paper completely characterizes poverty-reducing nonlinear tax schedules within an endogenous income framework accommodating heterogeneous wage distributions, individual-specific non-labor endowments, and idiosyncratic preferences. When negative

marginal tax rates are deemed infeasible, poverty-reducing tax schedules take a simple form: a fixed lump-sum subsidy for after-tax incomes below the poverty line. These schedules admit a complete preorder by the subsidy at zero income capped at the poverty line; subsidies at or above the poverty line are equivalent because the censored distribution has attained its maximal value. When negative marginal tax rates are permissible, a poverty-reducing schedule must have non-positive tax liabilities on $[0, z]$ and non-positive marginal tax rates over the poverty-relevant income range where net income remains strictly below the poverty line.

Several extensions clarify the scope and boundaries of these results, revealing robustness to poverty line uncertainty as well as to endogenous poverty lines anchored to predetermined reference statistics (e.g., lagged medians).

A comparison of our framework with its exogenous-income counterpart reveals how behavioral responses impose additional incentive-compatibility constraints. When income is fixed, virtually any subsidy to the poor suffices for poverty reduction. When income is endogenous, however, poverty-reducing schedules must have non-positive tax liabilities at gross incomes weakly below the poverty line and non-positive marginal tax rates wherever net income remains strictly below the poverty line. Uniform lump-sum subsidies eliminate substitution effects, whereas negative marginal tax rates may generate pro-work substitution effects.

The robustness of our characterization across preference domains containing the simple quasilinear functional forms used in the necessity proof stems from the Monotonicity axiom—embedded in censored first-order stochastic dominance—which treats income gains for the poor as unambiguously poverty-reducing. This Paretian principle enables the necessity argument to rely on homogeneous economies and requires only that the admissible preference domain contain the standard functional forms used to construct the counterexamples. By contrast, distribution-sensitive poverty measures (which replace monotonicity with inequality concerns among the poor) cannot leverage homogeneous counterexamples, as relative deprivation depends on heterogeneous behavioral responses. Consequently, poverty reduction under such criteria requires restrictions on preference elasticities—a limitation absent from our framework.

As discussed in [Section 4.2.5](#), heterogeneous behavioral responses create an obstruction to extending the universal poverty-reduction guarantee for uniform subsidies to $\succsim_R^z$ or $\succsim_A^z$ across fully heterogeneous preferences. Under the commonly adopted assumption of homogeneous preferences, however, this obstacle can disappear. In that case, uniform lump-sum subsidies—or more generally, marginal-rate progressive schedules—can guarantee poverty reduction under certain wage elasticity conditions. This aligns with results in the literature on taxation and income inequality, suggesting that structural assumptions on preferences could yield implementable, distribution-sensitive anti-poverty policies.

Finally, an open question is whether the analysis extends to multidimensional notions of poverty that consider health, education, housing, or environmental quality alongside income. Such an extension would require accounting for behavioral responses across multiple dimensions of deprivation.

A. Proof of Theorem 1

We begin with a result on the preorder $\succsim_D$:

Lemma 1. For any two income distributions $\mathbf{x} = (x_1, \dots, x_n)$ and $\mathbf{y} = (y_1, \dots, y_n)$, $x_i \geq y_i$ for each i implies $\mathbf{x} \succ_D \mathbf{y}$.

Proof. By definition of $\succ_D$, it suffices to show $x_{[k]} \geq y_{[k]}$ for all $k \in \{1, \dots, n\}$.

Suppose for contradiction that $x_{[k]} < y_{[k]}$ for some k . Define

$$A = \{i \in \{1, \dots, n\} : x_i \leq x_{[k]}\}.$$

By definition of the k -th order statistic, $|A| \geq k$. For each $i \in A$,

$$y_i \leq x_i \leq x_{[k]} < y_{[k]}.$$

Thus at least k elements of $\mathbf{y}$ are strictly less than $y_{[k]}$. But by definition of $y_{[k]}$ as the k -th smallest element, at most $k - 1$ elements of $\mathbf{y}$ can be strictly less than $y_{[k]}$ —a contradiction.

Hence $x_{[k]} \geq y_{[k]}$ for all k , so $\mathbf{x} \succ_D \mathbf{y}$. ■

We establish as a preliminary result that all poverty-reducing tax schedules necessarily involve poor subsidization.

Lemma 2. Suppose that $t \in \mathcal{T}$ is $\succ_z$ -poverty-reducing. Then $t(y) \leq 0$ for all $y \in [0, z]$.

Proof. Suppose, by contradiction, that $t(y_0) > 0$ for some $y_0 \in [0, z]$. Feasibility implies $t(0) \leq 0$, so $t(y_0) > 0$ requires $y_0 > 0$.

Choose a one-person economy with non-labor endowment $\omega = y_0$, any wage $a > 0$, and preferences

$$u(l, x) = x - Kl - \eta l^2,$$

where $K > a$ and $\eta > 0$. These preferences belong to $\mathcal{U}$: they are continuous, strictly increasing in consumption, strictly decreasing in labor, and strictly quasiconcave, and consumption is normal.

Under the tax-free schedule, utility at labor l relative to zero labor is

$$al - Kl - \eta l^2 < 0, \quad \text{for every } l > 0.$$

Thus, zero labor is uniquely optimal, and tax-free income is y_0 .

Under t , monotonicity implies $t(y_0 + al) \geq t(y_0)$. Hence, for every $l > 0$,

$$\begin{aligned} u(l, y_0 + al - t(y_0 + al)) - u(0, y_0 - t(y_0)) \\ = al - [t(y_0 + al) - t(y_0)] - Kl - \eta l^2 \leq al - Kl - \eta l^2 < 0. \end{aligned}$$

Therefore, zero labor is also uniquely optimal under t , and post-tax income is

$$y_0 - t(y_0) < y_0 \leq z.$$

In the one-person economy populated by this type, taxation strictly increases poverty, contradicting the assumption that t is $\succ_z$ -poverty-reducing. Therefore,

$$t(y) \leq 0, \quad \text{for every } y \in [0, z].$$

■

Theorem 1. A tax schedule $t \in \mathcal{T}$ is $\succsim_z$ -poverty-reducing if and only if, letting $s = -t(0)$, one has $s \geq 0$ and $t(y) = -s$ for every $y \in \mathbb{R}_+$ satisfying $y + s \leq z$.

Proof. Sufficiency. Let $s = -t(0) \geq 0$ and suppose $t(y) = -s$ whenever $y + s \leq z$. If $s \geq z$, rank preservation gives $y - t(y) \geq -t(0) = s \geq z$ for every $y \geq 0$, so the result is immediate. Suppose henceforth that $s < z$. We must show that t is $\succsim_z$ -poverty-reducing, meaning that for all endowment distributions $(\omega_1, \dots, \omega_n)$, all wage distributions $(a_1, \dots, a_n)$, all preference profiles $(u_1, \dots, u_n) \in \mathcal{U}^n$, and all solution functions $x_1^{u_1}, \dots, x_n^{u_n}$, we have

$$(x_1^{u_1}(\omega_1, a_1, 0), \dots, x_n^{u_n}(\omega_n, a_n, 0)) \succsim_z (x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t)). \quad (4)$$

For fixed parameters $(\omega_1, \dots, \omega_n)$, $(a_1, \dots, a_n)$, $(u_1, \dots, u_n)$, and solution functions $x_1^{u_1}, \dots, x_n^{u_n}$, let

$$c = (c_1, \dots, c_n) \quad \text{and} \quad c' = (c'_1, \dots, c'_n)$$

denote, respectively, the distributions

$$(x_1^{u_1}(\omega_1, a_1, 0), \dots, x_n^{u_n}(\omega_n, a_n, 0)) \quad \text{and} \quad (x_1^{u_1}(\omega_1, a_1, t), \dots, x_n^{u_n}(\omega_n, a_n, t))$$

censored at the poverty line z .

To see that (4) holds, it suffices to show that $c' \succsim_D c$, which is implied by the inequalities

$$c'_i \geq c_i, \quad i \in \{1, \dots, n\} \quad (5)$$

(Lemma 1).

To see that (5) holds, choose $i \in \{1, \dots, n\}$ and note that either $c'_i < z$ or $c'_i = z$.

The second case is easy to handle, since $c'_i = z \geq c_i$.

Suppose that $c'_i < z$, and let y'_i denote the gross income selected under t . Then $y'_i - t(y'_i) < z$. Rank preservation and $-t(0) = s$ imply $y - t(y) \geq s$ for every $y \geq 0$, so $y'_i + s < z$ and $t(y'_i) = -s$. Hence $c'_i = y'_i + s$.

Monotonicity of t and $t(0) = -s$ imply

$$y - t(y) \leq y + s \quad \text{for every } y \geq 0.$$

Thus the actual budget lies weakly below the global pure-subsidy line and coincides with it whenever $y + s \leq z$. Since y'_i is optimal under t , it maximizes the pure-subsidy objective over the initial interval on which $y + s \leq z$. Define

$$U_s(y) = u_i((y - \omega_i)/a_i, y + s).$$

The function U_s is quasiconcave on the feasible interval and strictly quasiconcave whenever the corresponding bundles lie in the interior of the consumption-labor domain. The former property follows at boundary points by continuity from strict quasiconcavity on the interior.

We claim that y'_i uniquely maximizes U_s over the entire feasible interval. Suppose, to the contrary, that some feasible $\tilde{y} > z - s$ satisfies $U_s(\tilde{y}) \geq U_s(y'_i)$. Because $y'_i < z - s$, choose two distinct points

$$y'_i < y_1 < y_2 < z - s.$$

Quasiconcavity and the fact that each y_k lies between y'_i and $\tilde{y}$ imply

$$U_s(y_k) \geq \min\{U_s(y'_i), U_s(\tilde{y})\} = U_s(y'_i), \quad k \in \{1, 2\}.$$

Since y'_i maximizes U_s over the initial subsidy region, equality must hold for both y_1 and y_2 . The bundles corresponding to y_1 and y_2 are interior. Strict quasiconcavity therefore implies

$$U_s(y) > U_s(y'_i), \quad \text{for every } y \in (y_1, y_2),$$

contradicting the optimality of y'_i over the initial subsidy region. Hence y'_i uniquely solves the global pure-subsidy problem

$$\max_{y_i \in [\omega_i, \omega_i + a_i]} u_i((y_i - \omega_i)/a_i, y_i + s).$$

Let y_i^* solve the corresponding tax-free problem. The tax-free budget and the global pure-subsidy budget are parallel linear budgets, with the latter obtained by an outward shift of size s . Normality of consumption therefore implies $y'_i + s \geq y_i^*$. Therefore $z > c'_i \geq c_i$, as required.

Necessity. Suppose that t is $\succsim_z$ -poverty-reducing. By [Lemma 2](#), $t(0) \leq 0$; let $s = -t(0) \geq 0$. If $s \geq z$, the condition is automatic at the boundary and the conclusion follows. Suppose henceforth that $s < z$. We must show that $t(y) = -s$ whenever $y + s \leq z$.

Suppose, by contradiction, that t does not have this property, and define

$$S = \{y \in [0, z] : y - t(y) \leq z\}.$$

Because $y - t(y)$ is continuous and nondecreasing, S is a closed initial interval containing 0. If every bracket intersecting the interior of S had marginal tax rate zero, continuity would imply $t(y) = t(0) = -s$ throughout S , and hence whenever $y + s \leq z$, contrary to the supposition. Therefore some bracket contains an interior point $\hat{y}$ of S at which $\hat{y} - t(\hat{y}) < z$ and has marginal tax rate $\tau > 0$.

Choose an interior poverty-relevant point $\hat{y}$ of a bracket with marginal tax rate $\tau > 0$, and choose $D > 0$ so that $I = [\hat{y}, \hat{y} + D]$ remains inside that bracket. By rank preservation, $\tau \leq 1$. Write

$$b = -t(\hat{y}) \geq 0.$$

Because the bracket is poverty-relevant, $\hat{y} + b < z$. Choose a wage a satisfying

$$b < a < z - \hat{y}.$$

Next choose $\varepsilon, \eta > 0$ such that

$$\varepsilon < \tau, \quad \varepsilon a < \tau D, \quad 2\eta a < \varepsilon.$$

Take non-labor endowment $\omega = \hat{y}$ and preferences

$$u(l, x) = x - a(1 - \varepsilon)l - \eta a^2 l^2.$$

This utility function belongs to $\mathcal{U}$: it is continuous, strictly increasing in consumption, strictly decreasing in labor, strictly quasiconcave, and consumption is normal.

Write $r = al$ for labor income and $q(y) = y - t(y)$ for net income. Under the tax-free schedule, utility relative to $r = 0$ is

$$r - ((1 - \varepsilon)r + \eta r^2) = \varepsilon r - \eta r^2.$$

Because $2\eta a < \varepsilon$, this expression is strictly increasing on $[0, a]$. Hence the unique tax-free choice is $r = a$, yielding net income $\widehat{y} + a$.

Under t , linearity on I gives

$$q(\widehat{y} + r) - q(\widehat{y}) = (1 - \tau)r \quad \text{for } 0 \leq r \leq D.$$

Thus, for every $r \in (0, D]$, the utility difference relative to $r = 0$ is

$$(1 - \tau)r - (1 - \varepsilon)r - \eta r^2 = (\varepsilon - \tau)r - \eta r^2 < 0.$$

For $r \geq D$, monotonicity of t and the increase τD in tax liability over I imply

$$q(\widehat{y} + r) - q(\widehat{y}) = r - (t(\widehat{y} + r) - t(\widehat{y})) \leq r - \tau D.$$

Therefore, for every $r \in [D, a]$, the utility difference relative to $r = 0$ is at most

$$r - \tau D - (1 - \varepsilon)r - \eta r^2 = \varepsilon r - \tau D - \eta r^2 < \varepsilon a - \tau D < 0.$$

It follows that the unique taxed choice is $r = 0$, yielding net income

$$q(\widehat{y}) = \widehat{y} - t(\widehat{y}) = \widehat{y} + b.$$

By the choice of a ,

$$x^u(\widehat{y}, a, 0) = \widehat{y} + a > \widehat{y} + b = x^u(\widehat{y}, a, t),$$

and both incomes are below z . The homogeneous economy populated by this type therefore contradicts poverty reduction. This completes the necessity proof. ■

B. Relaxing monotonicity

This section provides a complete characterization of poverty-reducing tax schedules in the absence of the monotonicity constraint, extending the results of [Theorem 1](#) to this more general setting.

For the remainder of this section, a *tax schedule* is defined as a continuous, piecewise linear, and rank-preserving function $t : \mathbb{R}_+ \rightarrow \mathbb{R}$ satisfying $t(y) \leq y$ for every $y \geq 0$, where $t(y)$ represents the tax liability corresponding to the pre-tax income level y .

Note that dropping the monotonicity assumption renders tax schedules flexible enough to accommodate negative marginal tax rates.

Let $\mathcal{T}^*$ denote the set of all tax schedules according to this weaker definition.

We first state and prove the analogue of [Lemma 2](#).

Lemma 3. *Suppose that $t \in \mathcal{T}^*$ is $\succ_z$ -poverty-reducing. Then $t(y) \leq 0$ for all $y \in [0, z]$.*

Proof. Suppose, by contradiction, that $t(y_0) > 0$ for some $y_0 \in [0, z]$. Feasibility implies $t(0) \leq 0$, so $y_0 > 0$.

Choose any wage $a > 0$ and let $q(y) = y - t(y)$. Because t is piecewise linear, q is Lipschitz on $[y_0, y_0 + a]$. Hence, there exists $L < +\infty$ such that

$$q(y_0 + al) - q(y_0) \leq Lal, \quad \text{for every } l \in [0, 1].$$

Choose $K > \max\{a, La\}$ and $\eta > 0$, and consider a one-person economy with non-labor endowment $\omega = y_0$, wage a , and preferences

$$u(l, x) = x - Kl - \eta l^2.$$

These preferences belong to $\mathcal{U}$: they are continuous, strictly increasing in consumption, strictly decreasing in labor, and strictly quasiconcave, and consumption is normal.

Under the tax-free schedule, utility at labor l relative to zero labor is

$$al - Kl - \eta l^2 < 0, \quad \text{for every } l > 0.$$

Thus, zero labor is uniquely optimal, and tax-free income is y_0 .

Under t , for every $l > 0$,

$$u(l, q(y_0 + al)) - u(0, q(y_0)) = q(y_0 + al) - q(y_0) - Kl - \eta l^2 \leq Lal - Kl - \eta l^2 < 0.$$

Therefore, zero labor is also uniquely optimal under t , and post-tax income is

$$y_0 - t(y_0) < y_0 \leq z.$$

Taxation therefore strictly increases poverty in this one-person economy, contradicting the assumption that t is $\succsim_z$ -poverty-reducing. Hence, $t(y) \leq 0$ for every $y \in [0, z]$. ■

Lemma 4. Let $t \in \mathcal{T}^*$ satisfy $t(y) \leq 0$ for every $y \in [0, z]$. Let $q(y) = y - t(y)$ and

$$S_z(t) = \{y \in [0, z] : q(y) < z\}.$$

At a kink, let q'_+ and q'_- denote the right-hand and left-hand derivatives of q , respectively. If t is not non-increasing on $S_z(t)$, there exists $\bar{y} \in S_z(t)$ satisfying one of the following:

- (i) $\bar{y}$ lies in the interior of a bracket on which $q'(\bar{y}) \in (0, 1)$;
- (ii) $\bar{y} > 0$ is a kink at which $q'_+(\bar{y}) = 0$ and $q'_-(\bar{y}) > 0$;
- (iii) $\bar{y} = 0$ and $q'_+(0) = 0$.

Proof. Because q is continuous and nondecreasing,

$$S_z(t) = \{y \in [0, z] : q(y) < z\}$$

is an initial interval, possibly relatively open at its upper endpoint.

Since t is not non-increasing on $S_z(t)$, there exist $x < y$ in $S_z(t)$ such that $t(x) < t(y)$. Because t is continuous and piecewise linear, some positive-length subinterval of $[x, y]$ lies in a bracket with marginal tax rate $\tau > 0$. Rank preservation implies $\tau \leq 1$, so

$$q' = 1 - \tau \in [0, 1)$$

on that bracket. If one such bracket has $\tau \in (0, 1)$, an interior point of its intersection with $S_z(t)$ satisfies (i).

Suppose instead that every such bracket has $\tau = 1$, so $q' = 0$ on each of them. Select one and let $[\bar{y}, y_B]$ be the maximal flat segment of q containing it, where possibly $y_B = +\infty$. The selected bracket contains a point at which $q < z$. Since q is constant on the maximal flat segment, $q(r) < z$ for every $r \in [\bar{y}, y_B]$, and hence $\bar{y} \in S_z(t)$.

If $\bar{y} = 0$, then $q'_+(0) = 0$, giving (iii). If $\bar{y} > 0$, maximality of the flat segment implies that the bracket immediately to its left does not have slope zero. Since rank preservation requires all slopes of q to be nonnegative,

$$q'_-(\bar{y}) > 0 = q'_+(\bar{y}),$$

giving (ii). ■

Lemma 5. *Let $t \in \mathcal{T}^*$ satisfy $t(y) \leq 0$ for every $y \in [0, z]$. If t is not non-increasing on $S_z(t)$, there exist a one-person economy (ω, a, u) and unique optimal gross incomes $\bar{y}$ under t and y^* under the tax-free schedule such that*

$$\bar{y} - t(\bar{y}) < y^* < z.$$

Proof. Let $\bar{y}$ be supplied by Lemma 4, and put

$$q(y) = y - t(y), \quad c = q(\bar{y}) < z, \quad m = q'_+(\bar{y}) \in [0, 1), \quad r = \frac{1+m}{2} \in (m, 1).$$

Set the non-labor endowment equal to $\omega = \bar{y}$. Hence, no gross income below $\bar{y}$ is feasible.

First, suppose that q is not constant on $[\bar{y}, +\infty)$. Choose $y_C > \bar{y}$ such that $q(y_C) > c$ and

$$q(y) - c \leq r(y - \bar{y}), \quad \text{for every } y \in [\bar{y}, y_C]. \quad (6)$$

Such a choice is possible. If $m > 0$, take y_C sufficiently close to $\bar{y}$ within the bracket immediately to its right, on which the slope of q is $m < r$. If $m = 0$, then q is initially constant. Since it is not constant on $[\bar{y}, +\infty)$, its maximal initial flat segment has a finite right endpoint, say y_B . As $y \downarrow y_B$ from the right,

$$\frac{q(y) - c}{y - \bar{y}} \rightarrow 0.$$

Taking $y_C > y_B$ sufficiently close to y_B , and using the monotonicity and piecewise linearity of q , yields (6) throughout $[\bar{y}, y_C]$.

Choose

$$y^* \in (c, \min\{z, q(y_C)\})$$

and a wage satisfying

$$a > \max\{y_C - \bar{y}, y^* - \bar{y}\}.$$

The feasible gross-income interval is then $[\bar{y}, \bar{y} + a]$, and both y_C and y^* belong to its interior. Let

$$M = \max_{y \in [\bar{y}, \bar{y} + a]} q'_+(y).$$

This number is finite because q is finitely piecewise linear on the compact feasible interval. Moreover, $M > 0$ because $q(y_C) > q(\bar{y})$.

Choose $\beta \in (r, 1)$ and $\epsilon \in (0, r/M)$. Select a continuously differentiable and strictly positive function F' satisfying

$$F''(x) < 0, \quad \text{for every } x \geq 0,$$

together with $F'(c) = 1$, $F'(y^*) = \beta$, and

$$F'(x) \leq \epsilon, \quad \text{for every } x \geq q(y_C).$$

Such a function exists because

$$c < y^* < q(y_C) \quad \text{and} \quad 0 < \epsilon < r < \beta < 1.$$

Let F be an antiderivative of F' .

Select a continuously differentiable function V' on $[\bar{y}, \bar{y} + a]$ satisfying $V''(y) > 0$ throughout the feasible interval, together with $V'(\bar{y}) = r$, $V'(y^*) = \beta$, and

$$V'(y) < \beta \quad \text{for } y < y^*, \quad V'(y) > \beta \quad \text{for } y > y^*.$$

Let V be an antiderivative of V' , and define

$$v(l) = V(\bar{y} + al), \quad u(l, x) = F(x) - v(l).$$

Under the tax-free schedule, the objective in gross-income space is $W_0(y) = F(y) - V(y)$. If $y < y^*$, then

$$F'(y) > F'(y^*) = \beta > V'(y),$$

whereas, if $y > y^*$,

$$F'(y) < F'(y^*) = \beta < V'(y).$$

Hence,

$$W'_0(y) > 0 \quad \text{for } y < y^*, \quad W'_0(y) < 0 \quad \text{for } y > y^*.$$

Therefore, y^* uniquely maximizes the tax-free objective.

Under t , the objective in gross-income space is

$$W_t(y) = F(q(y)) - V(y).$$

Consider first $y \in (\bar{y}, y_C]$. Since q is nondecreasing,

$$q(y) \geq q(\bar{y}) = c.$$

Moreover, F' is decreasing and $F'(c) = 1$, so $F'(x) \leq 1$ for every $x \geq c$. It follows from (6) that

$$F(q(y)) - F(c) \leq q(y) - c \leq r(y - \bar{y}).$$

Since V' is strictly increasing and $V'(\bar{y}) = r$,

$$V(y) - V(\bar{y}) > r(y - \bar{y}).$$

Therefore, $W_t(y) < W_t(\bar{y})$ for every $y \in (\bar{y}, y_C]$. In particular,

$$\Delta = V(y_C) - V(\bar{y}) - [F(q(y_C)) - F(c)] > 0.$$

Now consider any feasible $y > y_C$. Rank preservation and the definition of M imply

$$q(y) - q(y_C) \leq M(y - y_C).$$

Since $F'(x) \leq \epsilon$ for every $x \geq q(y_C)$, we have

$$F(q(y)) - F(q(y_C)) \leq \epsilon M(y - y_C).$$

Moreover, $V'(\hat{y}) > r$ for every feasible $\hat{y} > \bar{y}$. Hence,

$$V(y) - V(y_C) > r(y - y_C).$$

Consequently,

$$W_t(y) - W_t(\bar{y}) = -\Delta + F(q(y)) - F(q(y_C)) - [V(y) - V(y_C)] < -\Delta + (\epsilon M - r)(y - y_C) < 0.$$

Since no gross income below $\bar{y}$ is feasible, $\bar{y}$ is the unique taxed choice.

Second, suppose that $q(y) = c < z$ for every $y \geq \bar{y}$. Choose $y^* \in (c, z)$ and $a > y^* - \bar{y}$. Choose any continuously differentiable and strictly positive function F' satisfying $F''(x) < 0$ for every $x \geq 0$, and put $\beta = F'(y^*)$. Select a continuously differentiable function V' on $[\bar{y}, \bar{y} + a]$ satisfying $V''(y) > 0$ throughout the feasible interval, together with $0 < V'(\bar{y}) < \beta$, $V'(y^*) = \beta$, and

$$V'(y) < \beta \quad \text{for } y < y^*, \quad V'(y) > \beta \quad \text{for } y > y^*.$$

Defining F , V , v , and u as above, the same derivative comparison shows that y^* uniquely maximizes the tax-free objective. Under t , $W_t(y) = F(c) - V(y)$, which is strictly decreasing. Hence, $\bar{y}$ is the unique taxed choice.

It remains to verify that the constructed preferences belong to $\mathcal{U}$. In both cases, $F' > 0$, $F'' < 0$, $V' > 0$, $V'' > 0$. Hence, $v(l) = V(\bar{y} + al)$ is strictly increasing and strictly convex, and $u(l, x) = F(x) - v(l)$ is continuous, strictly increasing in consumption, strictly decreasing in labor, and strictly concave.

Consumption is normal. Fix a linear budget $x = e + wl$ with a nonempty feasible set. If $w = 0$, nonemptiness requires $e \geq 0$. Strict labor monotonicity then selects zero labor, independently of e , while selected consumption rises one-for-one with e . Suppose that $w > 0$. At an interior optimum, $wF'(x) = v'(l)$. Implicit differentiation gives

$$\frac{dx}{de} = \frac{v''(l)}{v''(l) - w^2 F''(x)} > 0.$$

If the same fixed endpoint of the labor interval remains optimal, selected consumption rises one-for-one with e . If instead the optimizer follows the moving lower feasibility boundary $l = -e/w$, selected consumption remains equal to zero. Strict concavity and the maximum theorem imply that the unique optimizer, and hence selected consumption, varies continuously with e . Selected consumption is locally nondecreasing in each of the preceding cases, and continuity prevents a downward jump when the optimizer moves

from one case to another. Thus, selected consumption is globally nondecreasing under every parallel outward shift.

Thus, $u \in \mathcal{U}$, and the constructed incomes satisfy

$$x^u(\bar{y}, a, t) = q(\bar{y}) = c < y^* = x^u(\bar{y}, a, 0) < z. \quad \blacksquare$$

Theorem 2. For $t \in \mathcal{T}^*$, let

$$S_z(t) = \{y \in [0, z] : y - t(y) < z\}.$$

A tax schedule $t \in \mathcal{T}^*$ is $\succsim_z$ -poverty-reducing if and only if $t(y) \leq 0$ for every $y \in [0, z]$ and t is non-increasing on $S_z(t)$.

A tax schedule that is non-increasing on an interval is automatically strictly rank-preserving on that interval.

Proof of Theorem 2. Sufficiency. Suppose that $t(y) \leq 0$ for every $y \in [0, z]$, and that t is non-increasing on $S_z(t)$. We show that every individual's censored net income weakly increases.

Fix an individual with primitives (ω_i, a_i, u_i) , and let y'_i and y_i^* denote, respectively, a selected gross income under t and the unique tax-free gross income. If $y'_i - t(y'_i) \geq z$, then censored post-tax income equals z , and the desired comparison is immediate. Suppose instead that

$$q(y'_i) = y'_i - t(y'_i) < z.$$

Then $y'_i < z$. Indeed, if $y'_i \geq z$, rank preservation and $t(z) \leq 0$ would imply

$$q(y'_i) \geq q(z) = z - t(z) \geq z,$$

a contradiction. Hence, $y'_i \in S_z(t)$. Setting $b = -t(y'_i) \geq 0$, we have $q(y'_i) = y'_i + b$.

Define the pure-subsidy objective

$$U_b(y) = u_i \left(\frac{y - \omega_i}{a_i}, y + b \right)$$

on $[\omega_i, \omega_i + a_i]$, and let $\hat{y}_i$ maximize it. This maximizer is unique. Indeed, continuity and strict quasiconcavity on the interior imply quasiconcavity on the full feasible interval. If two distinct gross incomes were maximizers, quasiconcavity would make every intermediate gross income a maximizer. Choosing two distinct intermediate points whose corresponding bundles lie in the interior would then contradict strict quasiconcavity there. Thus, $\hat{y}_i$ is the unique solution to

$$\max_{y \in [\omega_i, \omega_i + a_i]} U_b(y).$$

We claim that $\hat{y}_i \leq y'_i$. Suppose, to the contrary, that $\hat{y}_i > y'_i$.

First, suppose that $\hat{y}_i \in S_z(t)$. Since $S_z(t)$ is an initial interval and t is non-increasing on it, $t(\hat{y}_i) \leq t(y'_i)$. Therefore,

$$q(\hat{y}_i) = \hat{y}_i - t(\hat{y}_i) \geq \hat{y}_i - t(y'_i) = \hat{y}_i + b.$$

The actual bundle available at $\widehat{y}_i$ under t thus weakly dominates the pure-subsidy bundle at $\widehat{y}_i$. Since $\widehat{y}_i$ is the unique pure-subsidy maximizer, that pure-subsidy bundle is strictly preferred to the pure-subsidy bundle at y'_i . The latter coincides with the actual taxed bundle at y'_i . This contradicts the optimality of y'_i under t .

Now suppose that $\widehat{y}_i \notin S_z(t)$. Let $\bar{y}$ denote the upper endpoint of the nonempty initial interval $S_z(t)$. Then $y'_i < \bar{y} \leq \widehat{y}_i$. If $\bar{y} < \widehat{y}_i$, choose points satisfying

$$y'_i < y_1 < \bar{y} < y_2 < \widehat{y}_i.$$

Since $\widehat{y}_i$ is the unique pure-subsidy maximizer, $U_b(\widehat{y}_i) > U_b(y'_i)$. Quasiconcavity and the fact that each y_k lies between y'_i and $\widehat{y}_i$ imply

$$U_b(y_k) \geq \min\{U_b(y'_i), U_b(\widehat{y}_i)\} = U_b(y'_i), \quad k \in \{1, 2\}.$$

The bundles corresponding to y_1 and y_2 lie in the interior, and $\bar{y}$ lies strictly between them. Strict quasiconcavity on the interior therefore gives

$$U_b(\bar{y}) > \min\{U_b(y_1), U_b(y_2)\} \geq U_b(y'_i).$$

If $\bar{y} = \widehat{y}_i$, the same strict comparison follows directly from the unique optimality of $\widehat{y}_i$.

Moreover, taking limits from within $S_z(t)$ and using continuity of t gives $t(\bar{y}) \leq t(y'_i)$. Hence,

$$q(\bar{y}) = \bar{y} - t(\bar{y}) \geq \bar{y} - t(y'_i) = \bar{y} + b.$$

Thus, the actual bundle available at $\bar{y}$ under t weakly dominates the pure-subsidy bundle there. Since that pure-subsidy bundle is strictly preferred to the pure-subsidy bundle at y'_i , which coincides with the actual taxed bundle at y'_i , this again contradicts the optimality of y'_i under t . Therefore, $\widehat{y}_i \leq y'_i$.

It follows that

$$y'_i - t(y'_i) = y'_i + b \geq \widehat{y}_i + b.$$

The pure-subsidy budget is a parallel outward shift by b of the tax-free budget. Since consumption is normal, $\widehat{y}_i + b \geq y_i^*$. Consequently, $y'_i - t(y'_i) \geq y_i^*$.

Thus, every individual's censored post-tax income is weakly greater than their censored tax-free income. By [Lemma 1](#), the required $\succsim_z$ comparison follows. Therefore, t is $\succsim_z$ -poverty-reducing.

Necessity. Suppose that t is $\succsim_z$ -poverty-reducing. By [Lemma 3](#), $t(y) \leq 0$ for every $y \in [0, z]$. It remains to show that t is non-increasing on $S_z(t)$.

Suppose, by contradiction, that t is not non-increasing on $S_z(t)$. By [Lemma 5](#), there exist a one-person economy (ω, a, u) and unique optimal gross incomes $\bar{y}$ under t and y^* under the tax-free schedule such that $\bar{y} - t(\bar{y}) < y^* < z$. Thus, the post-tax and tax-free incomes both lie strictly below the poverty line, and post-tax censored income is strictly lower than tax-free censored income. This one-person economy therefore exhibits strictly greater poverty under t , contradicting the assumption that t is $\succsim_z$ -poverty-reducing. Hence, t is non-increasing on $S_z(t)$. ■

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